Comparative Analysis of Value at Risk (VaR) Methods for Portfolio with Non-Linear Return

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ABSTRACT

In this study various value at risk methods such as Historical Simulation, Variance–Covariance Approach and Monte Carlo Simulation are calculated, compared and tested for accuracy. Backtesting for the VaR methods is applied to check the accuracy of the VaR methods. The portfolio includes equally weighted three banking stock and one at-the-money (ATM) call option for one of the banking stock in the portfolio. The log return for the portfolio and individual investments are calculated. Different VaR calculation methods are used to calculate the downside risk of the portfolio and individual investments. VaR is calculated at 95% and 99% confidence level for the portfolio and individual securities. The value at risk for the portfolio at 95% confidence level from all the three methods are within the defined level of downside risk, while at 99% confidence level only Mote Carlo Simulation method provides good approximation of downside risk for a portfolio with options. Thus from this study it is inferred that for instrument or portfolio with non-linear return structure Monte Carlo simulation method provide good approximation of the downside risk.

Keywords: downside risk, Value at Risk, VaR, Historical Simulation, Variance–Covariance method, Monte Carlo Simulation method, Backtesting.

1. INTRODUCTION

The securities under study include only banking sector securities from Indian capital market. The data for the securities collected from 1st January 2004 to 31st December 2007 with the assumption of data stability for the sample period. The various method of VaR are tested at different level of significance and compared with each other as well as checked for the accuracy with actual data.

The securities from financial services sector specifically banking selected because the financial firms in most of the industrial countries performed strongly during the year 2007, and banks benefited from the credit environment and strong retail business.

Investment banks registered record profits due to growth in capital market and rise in the private equity.

According to the World Economic Outlook (WEO) of the International Monetary Fund (IMF) released in October 2007, global real GDP growth was expected to decline from 5.4 per cent in 2006 to 5.2 per cent in 2007 and further to 4.8 per cent in 2008. Slow growth in the United States in the first quarter of 2007 were seen, however it rebounded in the second and third quarters. Most of the other countries continued to expand strongly. The European and Japan, growth has remained above trend as their domestic demand is taking a more central role in the expansions. Also emerging market countries have continued to expand robustly, led by rapid growth in China, India and Russia.

The Indian banking sector booked high positive performance during 2006-2007 as given in Table 1. The faster growth of the banking sector in relation to the real economy pushed up the ratio of assets of scheduled commercial banks to GDP to 92.5 per cent at end-March 2007. The asset quality of Scheduled Commercial Banks improved during 2006-07, which can be seen from decline in gross and net non-performing assets as percentage of loans and advances.

Table 1. Performance of Banking Stocks Risk and Return

Indices	Returns*			Volatility@		
	2005-06	2006-07	2007-08#	2005-06	2006-07	2007-08#
BSE Bankex	36.8	24.3	72.7	11.8	17.5	13.8
BSE Sensex	73.7	15.9	52.5	16.7	11.1	12.0

*: Percentage Variation in indices on a point to point basis.

@: Defined as coefficient of variation.

#: Up to November 2007.

Source: RBI Report 2006-07.

1.1 Value at Risk: Value-at-risk (VaR) is defined as the maximum loss that a position or portfolio can suffer due to the market uncertainties with a certain confidence level and limited to certain time horizon. VaR models were originally introduced by US institutions (Citibank, J.P. Morgan, Chase Manhattan and Bankers Trust).

Therefore value at risk is a probabilistic measure, and takes different values at different confidence levels. If prob(E) indicates the probability of event E and c the confidence level, and L loss over the selected time horizon, the relationship is written as:

$$\text{prob}(L > \text{VaR}) = 1 - c \qquad (1)$$

In case of distribution of losses is discrete, VaR is defined as the smallest value v, such that the probability that losses will exceed v is no more than 1-c i.e.

$$\text{VaR} = \min \{v \mid \text{prob}(L>v) \leq 1-c\} \qquad (2)$$

In this study the confidence level used are 95% and 99% for each one day, five days and ten days value at risk.

1.2 VaR calculation Methods: Variety of methods exists for estimating VaR. Each method has its own set of assumptions, but the most common assumption is that historical market data is our best estimator for future changes. Various calculation methods include; Historical Simulation method, Variance-Covariance Method and Monte Carlo Simulation Method.

Within these calculation methodologies, risk measures, traditionally most widespread approach is the variance-covariance approach (also known as analytical or parametric method). Variance-Covariance method assumes that the possible change in the value of market factors (or, alternatively, of the returns of the assets in the portfolio) follow normal distribution. The information on the possible future values of market factors and their correlation is therefore entirely summarised in a variance-covariance matrix. Thus the possible losses on the portfolio depend on the matrix and on the sensitivity (which is usually approximated by a linear function with constant coefficients) of the individual positions in the portfolio to change in market factors.

2. LITERATURE REVIEW

There are lot of research work with different approach to calculate the VaR, testing and studies done on this topic. Institutions worldwide following value at risk as standard tool for risk management whether it is capital market, foreign exchange, fixed income or to evaluate the optimum portfolio or optimum trading size. The first VaR model was developed and published by US financial giant JP Morgan called RiskMetriicsTM methods in 1995. The RiskMetriicsTM is based on the variance covariance matrix. VaR methodology is also studied by Duffe and Pan (1997) and Jorion (1997) and provided various calculation methodologies.

The basic methodology to be followed by Banks is given by Resti and Andrea (2007). They present an integrated framework for risk measurement, capital management and value creation in banks. From the measurement of the risks faced by a bank, it defines criteria and rules to support a corporate policy aimed at maximizing shareholders' value. It discusses different risk types and how to assess the amount of capital they absorb by means of up-to-date, robust risk-measurement models. Also defined Value at Risk and calculation method i.e. Historical Simulation, Variance Covariance approach and Monte Carlo models.

Different methods mentioned above are also reviewed in detail from the “Value at Risk: The new benchmark for managing financial risk” by Philippe Jorion (2007). It provides the most current information needed to understand and implement VaR as well as manage newer dimensions of financial risk. It includes an increased emphasis on operational risk; using VaR for integrated risk management and to measure economic capital, applications of VaR to risk budgeting in investment management, discussion of new risk-management techniques, including extreme value theory, principal components and copulas. They discussed VaR from computing and backtesting models to forecasting risk and correlations. Also outlines the use of VaR to measure and control risk for trading, for investment management, and for enterprise-wide risk management. Also points out key pitfalls to watch out for in risk-management systems.

Coronado (1996) provides how to compare the different VaR calculation. The paper compares the different estimation methods of Value-at-Risk (VaR) as a market risk measurement of actual bank non-linear portfolios (specifically comprised of currency options) in the context of the supervision of bank solvency. In this paper author provides theoretical evidence as well as empirical to the precision of the Monte Carlo simulation methods to be preferred to the speed that can be obtained with the variance-covariance matrix method.

To evaluate the VaR methods Lopez (1999) in his document "Methods for Evaluating Value at Risk Estimates" describe that accuracy of VaR estimates is of the concern to bank and there regulators. In this paper two hypothesis-testing methods for evaluating VaR estimates have been proposed the binomial and interval forecast method. An alternative evaluation method, based on regulatory loss function which assign quadratic numerical scores when observed portfolio losses exceed VaR estimates.

Some new test of value at risk validation proposes by Hurlin and Tokpavi (2006). They uses the Multivariate Portmanteau statistic of Li and McLeod (1981) - extension to the multivariate framework of the test of Box and Pierce (1970) - to jointly test the absence of autocorrelation in the vector of Hit sequences for various coverage rates considered as relevant for the management of extreme risks. Then they showed that this shift to a multivariate dimension appreciably improves the power properties of the VaR validation test for reasonable sample size.

3. DATA COLLECTION AND METHODOLOGY

3.1 Data Collection: The data for the study i.e. for the securities (ICICI Bank, Kotak Bank, SBI Bank and ICICI Call option ATM) in the portfolio to be analysed collected from the National Stock Exchange (NSE), Mumbai. The data is collected for the period 1st January 2004 to 31st December 2007 i.e. for four years which is equal to 1004 trading days. The data is combined into one data table on basis of common trading date for the period. The portfolio assumed to be equally weighted security portfolio for the simplification.

The daily log returns of securities calculated for the sample period. The returns are calculated as follow:

$$R_t = \ln(S_t/S_{t-1}) \approx \Delta S_t/S_{t-1} \qquad (3)$$

Where St is the value of the security at time t. Based upon these data, mean and standard deviation can be estimated. The average return for the portfolio at time t can be calculated as average of the log return of the securities in the portfolio.

$$R_{pt} = \left(\sum_{i=1}^{4} Rit\right)/4 \qquad (4)$$

Where Rpt is the return of portfolio at time t and i is the ith security.

3.2 Historical Simulation Method: This is the simplest method of calculating VaR. This method does not require the assumption of normal distribution of securities return. The distribution of risk factor changes is assumed to be stable over time, so that their past behavior is a reliable guidance to predict their possible future movements.

3.3 Variance Covariance Method: This method assumes that stock returns are normally distributed. It requires estimating only two factors expected return and a standard deviation, which allow plotting normal distribution curve.

The normal distribution is widely used to describe random movement, and is characterised by two parameters only, mean and standard deviation. It is represented as follows:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\left(\frac{x-\mu}{2\sigma}\right)^2} \qquad (5)$$

where f(x) is the density function, μ the mean and σ the standard deviation. From this can calculate the standard normal distribution (i.e., that particular normal distribution which is characterized of by a mean of zero and standard deviation of one). Using the standard normal cumulative density function is advantageous as this no longer depend on μ the mean and σ the standard deviation, but depend only on parameter α (= $(x - \mu)/\sigma$). A precise link between different values of α and the corresponding probability levels can therefore be established, and remains valid regardless of the values taken by the mean and standard deviation of the variable considered.

The assumption of a normal distribution of returns therefore allows to convert a selected probability level into an appropriate scaling factor α, to which a threshold u – represented by the mean plus α times the standard deviation. As the portfolio holds long position in the securities, the value of the α will be selected in such a way as to isolate the left tail of the distribution.

$$u = \mu + \alpha.\sigma \qquad (6)$$

The variance - covariance approach is often used assuming that market factor returns have zero mean. The empirical studies show that the best prediction of the future return is not the historical mean return, but rather a value of zero.

If an expected return μ of zero is assumed, then:

$$u = \alpha . \sigma \qquad (7)$$

when, eqn. (7) multiplied by the market value (MV) of the portfolio at risk, will result in value at risk as follow:

$$\text{VaR} = \text{MV} \cdot \alpha \cdot \sigma \qquad (8)$$

Thus VaR of a position of the portfolio is the product of three elements: 1) market value (MV) of the position, 2) the relevant market factor's estimated return volatility (σ), 3) a scaling factor α of the hypothesis of a normal distribution of market factor returns allows to obtain a risk measure corresponding to the desired confidence level.

The above VaR is for daily position of the portfolio. To calculate VaR for longer time horizon need to estimate the volatility for longer time horizon. If daily returns *r* are assumed to be represented by independent and identically distributed random variable, with a mean of r and a variance of σ_r^2, then the return for a period of T days, $R_T = \sum_{t=1}^{T} R_t$ is also normally distributed, with a mean of T, $\bar{r}$ and variance of T, σ_r^2. Thus standard deviation σ_T for the period T can therefore be obtained from the daily σ_D as:

$$\sigma_T = \sigma_D \sqrt{T} \qquad (9)$$

The overall VaR of the portfolio is less than the sum of VaR of each security as follow:

$$VaR_P \leq \sum_{i=1}^{N} VaR_i \qquad (10)$$

VaR calculated using the parametric approach is therefore a subadditive risk measure, i.e. when multiple positions are combined, the total risk measured through VaR can only be lower, then the sum of individual positions risk. So, this risk measure correctly implements the risk diversification principle.

Consider a portfolio comprising N positions, characterised by value at risk of VaR_1, $VaR_2,\ldots,VaR_N$ respectively. The value at risk of the individual positions in vector form as:

$$\boldsymbol{v} = \begin{bmatrix} VaR_1 \\ VaR_2 \\ \ldots \\ VaR_N \end{bmatrix} \qquad (11.1)$$

Similarly, correlation coefficients among market factor returns can be expressed combined as follow:

$$\mathbf{C} = \begin{bmatrix} 1 & \rho_{1,2} & \ldots & \rho_{1,N} \\ \rho_{2,1} & 1 & \ldots & \rho_{2,N} \\ \ldots & \ldots & \ldots & \ldots \\ \rho_{N,1} & \ldots & \ldots & 1 \end{bmatrix} \qquad (11.2)$$

Thus, value at risk of the portfolio is given by:

$$VaR_P = \sqrt{v' \cdot C \cdot v} \quad \text{, where } v' \text{ is transpose of } v, \qquad (11.3)$$

3.4 Monte Carlo Simulations: The Monte Carlo Simulation method involves developing a model for future stock price returns and running multiple hypothetical trials through the model. It is based on the generation of random data, but through a more complex mechanism. It involves the estimation of the parameters of a probability distribution (e.g. normal, Student t-distribution etc.) from the historical sample, and then the extraction of N simulated values for the risk factor(s) from this probability distribution. Thus this technique allows generating a number of values which may even be larger than the number of observations in the historical sample.

Monte Carlo simulations in finance used as a pricing tool for the complex products, such as derivative and some exotic options. The application of Monte Carlo simulation to risk management is same as the one used in pricing i.e. from an appropriate parameterized theoretical distribution, the evolution of a market variable will be simulated a large number of times, and the market value of the individual risk position at each of the risk scenarios will be calculated.

4. COMPARISON OF VAR METHODS

The Distribution of financial instruments return in case of Variance – Covariance Method is Normal, in case of historical simulation is Stationary (Historical) and in case of Monte Carlo simulation method is Completely flexible.

The value at a given confidence level in case of Variance Covariance Method is a multiple of the standard deviation in case of historical simulation is percentile of the distribution of portfolio value changes while in case of Monte Carlo simulation is percentage of the simulated distribution of portfolio value changes. The change in positions market values in case of Variance – Covariance method is approximated by either linear functions or quadratic functions while in case of historical simulation and Mote Carlo simulation is calculated from the new market conditions ("full valuation"), even though linear approximations can also be used. The interaction among multiple markets factors in case of Variance – Covariance method is calculated through a correlation matrix in case of historical simulation is implied in the historical distribution while in case of Mote Carlo simulation is calculated through a correlation matrix and Cholesky's decomposition method.

The merits of these three methods i.e. Variance – Covariance method is a fast computing method. It does not require a pricing model for each position. In case of historical simulation method, it does not require explicit hypotheses about risk factor distribution, volatility and correlation estimates (it preserve past ones). The Monte Carlo simulation method can be used for complex portfolio and totally flexible distribution of market factors.

These methods also have some pre assumption or drawback to use. Variance – Covariance method assumes that the data is normally distributed and requires an explicit volatility and correlation estimates. This method also follows that payoff are linear. The historical simulation method requires large historical sample data and requires a pricing model for each position. The Monte Carlo simulation method is computationally intensive and requires pricing model for each position.

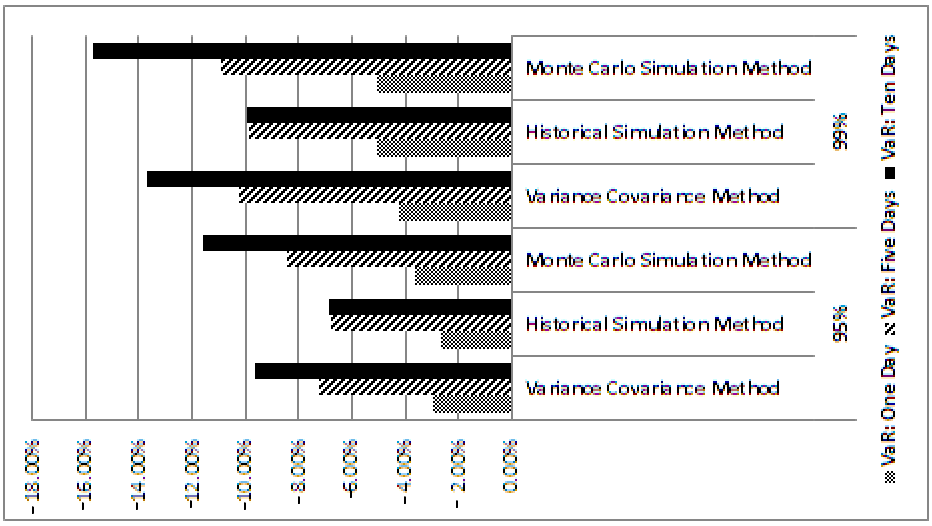
In case the portfolio has large position characterized by linear payoffs in the portfolio the Variance – Covariance approach nevertheless has the merit of being simple, rapidly computable and responsive to possible increases in risk factor volatility. In case the portfolio has a high amount position characterized by non-linear payoffs (such as options) full valuation (historical or Monte Carlo simulation) models are likely to be more adequate.

Table 2. VaR from different method at different confidence level and duration

Confidence Level	95%			99%		
	Variance Covariance Method	Historical Simulation Method	Monte Carlo Simulation Method	Variance Covariance Method	Historical Simulation Method	Monte Carlo Simulation Method
VaR: One Day	-3.014%	-2.715%	-3.695%	-4.262%	-5.058%	-5.099%
VaR: Five Days	-7.246%	-6.793%	-8.464%	-10.245%	-9.896%	-10.936%
VaR: Ten Days	-9.662%	-6.891%	-11.570%	-13.662%	-9.952%	-15.723%

The VaR value for one day, five days and ten days as give in the Table 2, the VaR value is least for the historical simulation method and highest for Monte Carlo Method as from Table 2.

Figure 1. VaR for different method at different confidence level and duration



The variance – covariance understates the VaR return at the given level of significance for the given duration. While Monte Carlo method overstate the VaR return value for the portfolio under study with stated parameters as given in Table 2 and Figure 1.

5. EVALUATION OF THE VAR METHODS - BACKTESTING

The backtesting is based on a comparison between the methods indications and trading results i.e. a comparison between daily estimated VaR and the actual losses for the following day. The underlying logic is relatively simple: if the method is correct, actual losses should exceed VaR with a frequency that is consistent with the one defined by the confidence level.

5.1 One day VaR analysis

The daily VaR of the portfolio under study is -4.26% (Table 2) changes in the portfolio value at 99% confidence level using variance covariance method. The total numbers of daily returns are one thousand three only for the data under study, 1% of the one thousand three is ten observations of the daily returns. Thus likely to expect losses in excess of -4.26% in the portfolio value in 1% of the cases i.e. ten observations. The actual percentage loss for the portfolio value exceeding -4.26% of the portfolio value recorded in fifteen (Table 3 and Figure 2) observations from the sample. The number of days on which losses exceed -4.26% of the portfolio value is significantly different from the from the confidence level's predictions, thus method is likely to be inadequate.

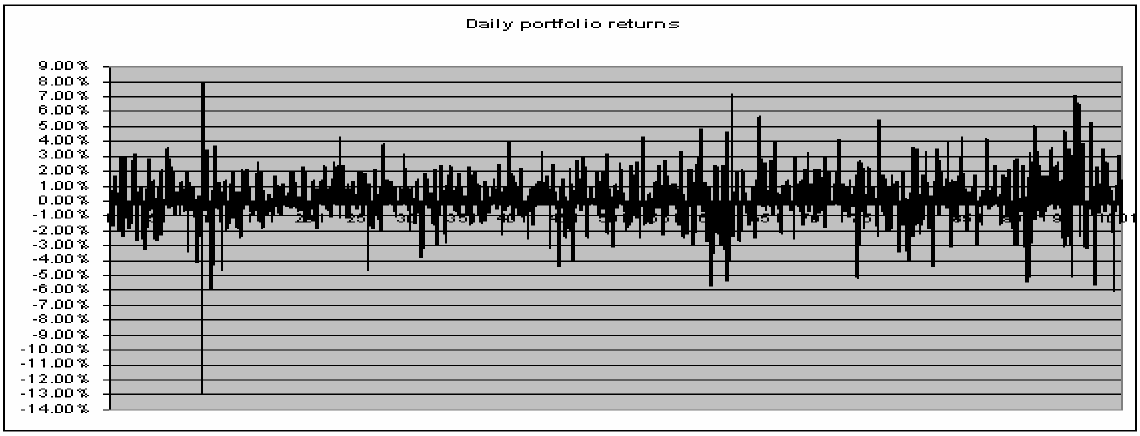
The daily VaR of the portfolio under study is -5.06% (Table 2) changes in the portfolio value at 99% confidence level using historical simulation method. The total numbers of daily returns are one thousand three only for the data under study, 1% of the one thousand three is ten observations of the daily returns. Thus likely to expect losses in excess of -5.06% in the portfolio value in 1% of the cases i.e. ten observations. For this case the actual loss for the portfolio value exceeding -5.06% of the portfolio value recorded in eleven observations (Table 3 and Figure 2) from the sample. The number of days on which losses exceed -5.06% of the portfolio value is not significantly different from the confidence level's predictions, the method is likely to be adequate.

Table 3. Number of observation Exceed VaR at 95% and 99% confidence level for daily return data

	Number of observation exceed VaR at 95%	%age of observation exceed VaR at 95%	Number of observation exceed VaR at 99%	%age of observation exceed VaR at 99%
Variance Covariance Method	39	3.89%	15	1.50%
Historical Simulation Method	50	4.99%	11	1.10%
Monte Carlo Simulation	22	2.19%	10	1.00%

The daily VaR of the portfolio under study is -5.10% (Table 2) changes in the portfolio value at 99% confidence level using Monte Carlo Simulation method. The total number of daily returns is one thousand three for the data under study, 1% of the one thousand three is approx. ten observations of the daily returns. Thus likely to expect losses in excess of -5.10% in the portfolio value in 1% of the cases i.e. ten observations. For this case the actual loss for the portfolio value exceeding -5.10% of the portfolio value in exactly ten sample observations. The number of days on which losses exceed -5.10% of the portfolio value is almost equal from the confidence level's predictions, the method is likely to be adequate than other two methods. At 95% confidence level all the methods are almost adequate; this can be judged from the Table 3 and Figure 2.

Figure 2. Actual daily return data for whole duration



5.2 Five day VaR analysis

The five day VaR of the portfolio under study is -7.246% (Table 2) changes in the portfolio value at 95% confidence level using variance covariance method. The total sample size for five days returns is two hundred for the data under study, 5% of the two hundred is ten observations of the five day returns. Thus likely to expect losses in excess of -7.246% in the portfolio value in 5% of the cases i.e. ten observations. For this case the actual loss for the portfolio value exceeding -7.246% of the portfolio value recorded for 7 observations (Table 4).

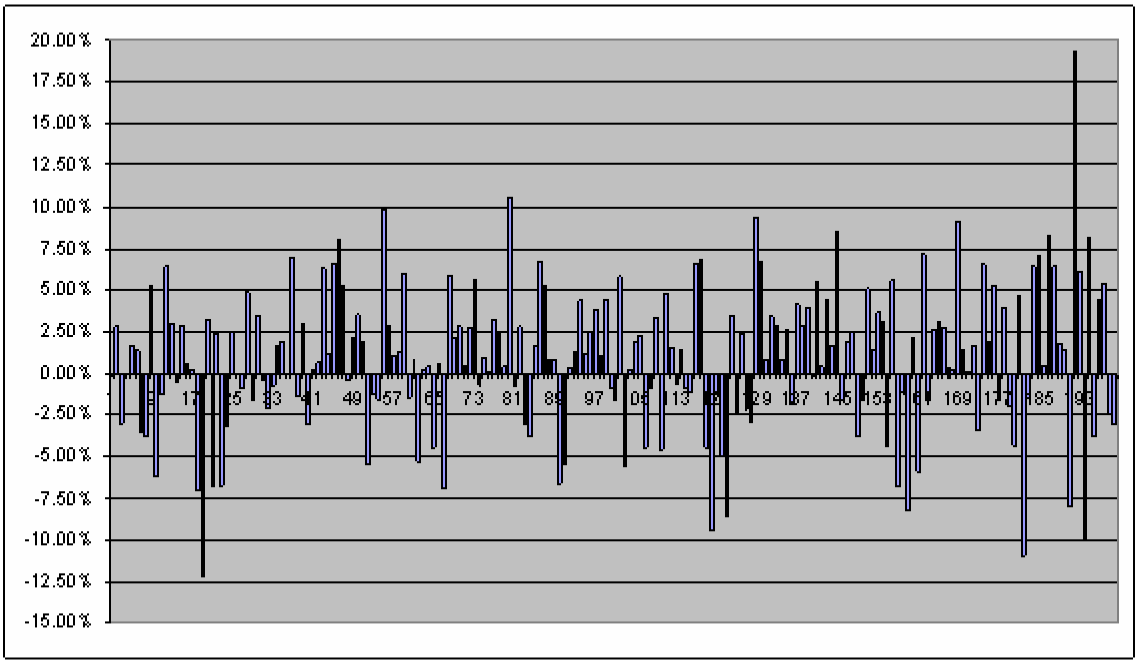
Table 4. Number of observation Exceed VaR at 95% and 99% confidence level for Five days return data

	Number of observation exceed VaR at 95%	%age of observation exceed VaR at 95%	Number of observation exceed VaR at 99%	%age of observation exceed VaR at 99%
Variance Covariance Method	7	3.50%	2	1.00%
Historical Simulation Method	10	5.00%	2	1.00%
Monte Carlo Simulation	5	2.50%	2	1.00%

The number of days on which losses exceed -7.246% of the portfolio value is less than from the from the confidence level's predictions, the method is likely to be adequate. At 99% confidence level the number of days on which losses exceed -10.245% of the portfolio value is equal to the confidence level's predictions, thus the method is likely to be adequate for five day VaR (Table 4 and Figure 3).

However, Monte Carlo simulation provides more adequate results than historical and variance covariance simulation method as given in the Table 4 and Figure 3.

Figure 3. Actual five days return data for whole duration



5.3 Ten day VaR analysis

The ten day VaR of the portfolio under study is -11.570% (Table 2) changes in the portfolio value at 95% confidence level using Monte Carlo Simulation method. The total number of five days returns are 100 for the data under study, 5% of the 100 is 5 observations of the ten day returns. Thus likely to expect losses in excess of -11.570% in the portfolio value in 5% of the cases i.e. 5 observations.

Table 5. Number of observation Exceed VaR at 95% and 99% confidence level for Ten days return data

	Number of observation exceed VaR at 95%	%age of observation exceed VaR at 95%	Number of observation exceed VaR at 99%	%age of observation exceed VaR at 99%
Variance Covariance Method	2	2.00%	1	1.00%
Historical Simulation Method	5	5.00%	1	1.00%
Monte Carlo Simulation	1	1.00%	0	0.00%

The actual loss for the portfolio value exceeds -11.570% (Table 2) for 1 observation (Table 5) only. The number of days on which losses exceed -11.570% of the portfolio value is less than from the from the confidence level's predictions, the method is likely to be adequate. At 99% confidence level the number of days on which losses exceed -15.723% of the portfolio value is less than the confidence level's predictions, thus the method is adequate

The historical simulation and variance covariance methods also provide almost adequate results at 95% and 99% confidence level, however Monte Carlo Simulation results are more adequate as given in the Table 5.

However apparently simple, backtesting VaR methods poses numerous problems, and can follow different logics. There are numerous alternative model have been proposed to evaluate the accuracy of a VaR method.

These alternative VaR evaluation models can be divided into three main categories: (i) tests based upon the frequency of exceptions i.e. the unconditional coverage test and the conditional coverage test. (ii) tests based upon a loss function i.e. the Lopez test based upon a loss function. (iii) tests based upon the entire profit and loss distribution.

6. CONCLUSION

The portfolio under study includes four banking securities from the Indian capital market. The securities are ICICI bank, Kotak bank, State bank of India and a call option on ICICI bank. The value at risk is calculated at 95% and 99% for different duration, one day VaR, five day VaR and ten days VaR for the individual securities as well as portfolio. The value at risk is calculated using three methods Historical Method, Variance Covariance Method and Monte Carlo Simulation at certain confidence level and time horizon.

The daily returns data Value at Risk at 95% confidence level is -2.715% (Table 2) for the portfolio daily return using Historical simulation method. Similarly five days and ten days Value at Risk at 95% confidence level is -6.793% and -6.896% (Table 2) respectively. The daily returns data Value at Risk at 99% confidence level is -5.058% for the portfolio value. Similarly, five days and ten days Value at Risk at 99% confidence level is -9.896% and -9.952% (Table 2) respectively.

The Value at Risk using variance – covariance approach at 95% confidence level is -3.014% (Table 2) for the portfolio daily return. Similarly the value at risk for five days and ten days return data 95% confidence level is -7.246% and -9.662% (Table 2) respectively for the portfolios five days and ten days returns. The value at risk using variance covariance approach at 99% confidence level is -4.262% (Table 2), i.e. the largest negative change in the portfolio daily return. Similarly the value at risk for five days and ten days return data at 99% confidence level is -10.245% and -13.662% (Table 2) respectively for the portfolios five days and ten days returns.

The value at risk for the portfolio using Monte Carlo simulation at 95% and 99% confidence level of significance is -3.695% and -5.099% (Table 2) respectively. The VaR for five days return and ten days return at 95% and 99% confidence level are -8.464 and -10.936, -11.570 and -15.7235 respectively.

For the portfolio having position characterized by non-linear payoffs (such as options) full valuation methods i.e. Monte Carlo simulation methods are likely to be more adequate than variance covariance method.

Table 6. Comparison of VaR methods for daily returns

	%age of observation exceed VaR at 95%	%age of observation exceed VaR at 99%
Variance Covariance Method	3.89%	1.50%
Historical Simulation Method	4.99%	1.10%
Monte Carlo Simulation	2.19%	1.00%

From Table 6 for the portfolio under study at 95% confidence level Monte Carlo simulation is more adequate than historical simulation and variance covariance method. At 99% confidence level variance covariance method is inadequate as portfolio includes one non-linear payoff security (Call option). Also the results from historical are inadequate at 99% confidence level.

The Monte Carlo simulation method provides the most adequate result at 95% confidence level as well as 99% confidence level, when portfolio include non-linear payoff security.

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