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Jump Risks and the Intertemporal Capital Asset Pricing Model

This paper investigates an economy where stock prices follow a jump-diffusion process. The first part of the analysis derives a sufficient condition under which an instantaneous CAPM will characterize equilibrium expected returns. The sufficient condition is that the jump component of a stock's return must be "unsystematic" and diversifiable in the market portfolio. The second part of the paper is an empirical investigation of the satisfaction of this condition. The evidence is seen to be inconsistent with the satisfaction of this hypothesis, that is, the market portfolio appears to contain a jump component.

I. Introduction

The specification of the stochastic process for stock prices is an important assumption underlying most equilibrium and option-pricing models in modern finance. This is particularly true of the continuous-time capital asset pricing models where it is assumed that asset prices follow diffusion processes with continuous sample paths (see Merton 1973*a*; Breeden 1979; Grossman and Shiller 1982). The extent to which the implications of these models generalize to discontinuous sample paths is an unanswered question in theoretical finance. However, the answer to this question is of more than just theoretical interest since accumulated empirical evidence appears to be inconsistent with the continuous sample path assumption (see Rosenberg 1972; Oldfield, Rogalski, and Jarrow 1977; Rosenfeld 1982).

One purpose of this paper is to provide some answers to this question. We do this by extending Merton's (1973*a*) intertemporal asset pricing model, in the special case of a constant investment opportunity set, to include discontinuous sample paths for asset prices. The constant investment opportunity set assumption is imposed because we are interested in obtaining sufficient conditions under which an instantaneous capital asset pricing model (CAPM) results. The extension of our model to a stochastic opportunity set is discussed briefly in the text.

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A sufficient condition which implies an instantaneous CAPM is derived under discontinuous sample paths. Loosely speaking, this sufficient condition is that the jump component of an asset's return is diversifiable in the market portfolio. This sufficient condition is similar to that obtained by Merton (1976) using Ross's (1976) arbitrage pricing model. Our analysis differs from his in that we consider an economy with a finite number of assets. Under this sufficient condition, jump risk does not receive "compensation" in terms of expected return in equilibrium since it is diversifiable. The second part of our paper is an empirical test designed to determine whether jump risk is "diversifiable." We do this by testing whether the market portfolio contains a jump component.

The remainder of our paper is organized as follows. The next section, Section II, generalizes the instantaneous CAPM to include discontinuous sample paths. Section III empirically tests for the existence of jump components in the market portfolio, and Section IV presents a conclusion.

II. Theory

The following economy is similar to that contained in Merton (1973*a*). It is a pure exchange model with a finite number of assets and traders. There is one consumption good which serves as numeraire. The basic assumptions are:

1. No transaction costs, no taxes, unrestricted short sales, and infinitely divisible asset shares (frictionless markets).
2. Traders act as price takers (competitive markets).
3. Trading takes place continuously at equilibrium prices.
4. There is a market for instantaneous borrowing and lending at a risk-free rate r.
5. There are n risky assets whose prices satisfy[1]

$$\frac{dS_j}{S_j} = \alpha_j dt + \sigma_j dZ_j + (-\lambda_j K_j dt + \pi_j dY_j) \quad j = 1, \ldots, n, \qquad (1)$$

where $S_j(t)$ is the price of asset j at time t; α_j, σ_j, λ_j, and K_j are constants; dZ_j is a Wiener process; dY_j is a Poisson process with parameter λ_j; π_j is the jump amplitude with expected value equal to K_j; and dZ_j, dY_j, and π_j are independent.

6. Traders have homogeneous beliefs over $\{\alpha_j, \sigma_j, \lambda_j, K_j, j = 1, \ldots, n\}$.
7. Traders' preferences are represented by $E_0\{\int_0^T U[c(t), t]dt + B[W(T), T]\}$, where $c(t)$ is consumption at time t, U is a von Neumann–

1. The random variable π_j must satisfy certain technical restrictions in order that a solution to (1) exists, see Kushner (1967).

Morgenstern utility function which is strictly increasing and strictly concave in $c(t)$ and twice differentiable, B is strictly increasing and strictly concave in $W(T)$, $W(T)$ is wealth at time T, and $E_0(\cdot)$ is the expectation operator conditional upon the information available at time 0.

Assumptions 1–4, 6, and 7 are standard in the literature; see Merton (1973*a*) for a detailed discussion. Assumption 5 is the key assumption in our analysis.

It is convenient at this time to rewrite assumption 5 in an alternative but equivalent form which isolates systematic and unsystematic risk components. Consider the diffusion part of assumption 5,

$$dD_j = \alpha_j dt + \sigma_j dZ_j, \quad j = 1, \ldots, n. \tag{2}$$

Using an analogous argument to Ross (1978, p. 272), expression (2) implies that there exists $\{\phi_j, f_j, g_j, d\psi, d\eta_j\}\, j = 1, \ldots, n$, such that

$$dD_j = \alpha_j dt + f_j d\psi + g_j d\eta_j, \tag{3a}$$

where $f_j^2 + g_j^2 = \sigma_j^2$; $d\psi$, $d\eta_j$ are Wiener processes; $E(d\psi d\eta_j) = 0, j = 1, \ldots, n$; and

$$\sum_{j=1}^{n} \phi_j = 1, \sum_{j=1}^{n} \phi_j(g_j d\eta_j) = 0, \sum_{j=1}^{n} \phi_j\alpha_j > r. \tag{3b}$$

It is always possible to decompose a finite number of normal random variables into a common factor, $d\psi$, and residuals, $d\eta_j$, which themselves are normally distributed. The key property of normal returns utilized is that a covariance of zero implies statistical independence. This same argument is contained in Fama (1973). Note that $d\psi$, $d\eta_i$ will be independent of dY_i and π_i by assumption 5. This decomposition gives $d\psi$ the interpretation of being the systematic risk factor and $d\eta_j$ the interpretation of being the unsystematic risk factor. Substitution of expression (3) into (1) gives assumption 5a: There are n risky assets whose prices satisfy

$$\frac{dS_j}{S_j} = \alpha_j dt + f_j d\psi + g_j d\eta_j + (-\lambda K_j dt + \pi_j dY_j), \quad j = 1, \ldots, n, \tag{4}$$

where $S_j(t)$ is the price of asset j at time t; $\alpha_j, f_j, g_j, \lambda_j, K_j$ are constants; $d\psi$, $d\eta_j$ are Wiener processes; dY_j is a Poisson process with parameter λ_j; π_j is the jump amplitude with expected value equal to K_j; and $d\psi$, $d\eta_j$, dY_j, π_j are independent.

The jump component in expression (4), $(-\lambda_j K_j dt + \pi_j dY_j)$, implies that asset returns can have discontinuous sample paths. This generalizes existing models. However, we also assume a constant (deterministic) investment opportunity set. This part of the assumption can be slightly weakened to any condition which implies that the derived utility of wealth function is state independent. This is discussed later in the text.

Under assumptions 1–7, the trader's consumption/portfolio decision is given by

$$\max_{\{c(t), w_j(t) \text{ for all } j\}} E_0 \left\{ \int_0^T U[c(t), t]dt + B[W(T), T] \right\}$$

subject to

$$W(t + \delta) = W(t) + \int_t^{t+\delta} \left\{ \sum_{j=1}^{n} w_j(x) W(x) [dS_j(x)/S_j(x) - r(x)dx] \right\} + \int_t^{t+\delta} [r(x)W(x) - c(x)]dx \tag{5}$$

and $W(0) \equiv W_0$, where $w_j(x)$ is the percentage of the risky asset portfolio held in asset j at time x, $j = 1, \ldots, n$.

The main theorem of this section gives a sufficient condition for two-fund separation and therefore an instantaneous CAPM.[2] The sufficient condition is that the jump component is "nonsystematic" risk and diversifiable. Although not done here, this theorem could be generalized to consider multiple fund separation. Let the market portfolio's weights be given by $(m_1, \ldots, m_n)$, where $\Sigma_{j=1}^{n} m_j = 1$.

THEOREM: Given assumptions 1–7, if

$$\sum_{i=1}^{n} m_i(g_i d\eta_i - \lambda_i K_i dt + \pi_i dY_i) = 0 \tag{6}$$

and $\Sigma_{i=1}^{n} m_i \alpha_i > r$, then $\alpha_i - r = \beta_i(\alpha_m - r)$, where $\alpha_m dt = E_t(\Sigma_{j=1}^{n} m_j dS_j/S_j)$ and

$$\beta_i = \text{cov}_t\left(dS_i/S_i, \sum_{i=1}^{n} m_i dS_i/S_i\right) \Big/ \text{var}\left(\sum_{i=1}^{n} m_i dS_i/S_i\right).$$

PROOF: Let us solve the dynamic stochastic programming problem given in expression (5). It is convenient to define an indirect utility (or Bellman) function, J, by

$$J[W(t), t] \equiv \max_{[c(t), w_j(t)]} E_t \left\{ \int_t^T U[c(t), t]dt + B[W(T), T] \right\}. \tag{7}$$

2. We define two-fund separation as follows (see Ross 1978): $(r, dS_1/S_1, \ldots, dS_n/S_n)$ exhibits two-fund separation if there exist two mutual funds: $(\beta_0, \ldots, \beta_n)$, $\Sigma_{i=0}^{n} \beta_0 = 1$ and $(\psi_0, \ldots, \psi_n)$, $\Sigma_{i=0}^{n} \psi_i = 1$ such that for all other portfolios $(\gamma_0, \ldots, \gamma_n)$, $\Sigma_{i=1}^{n} \gamma_i = 1$ and for all von Neumann–Morgenstern utility functions, v, which are increasing and concave, there exists (c_1, c_2) such that $\delta_i = c_1\psi_i + c_2\beta_i$, where $i = 0, \ldots, n$, $c_1 + c_2 = 1$, and

$$E\left\{v\left(\delta_0 r + \sum_{j=1}^{n} \delta_j dS_j/S_j\right)\right\} \geq E\left\{v\left(\gamma_0 r + \sum_{j=1}^{n} \gamma_j dS_j/S_j\right)\right\}.$$

We assume the existence of both the optimal controls satisfying (5) and the existence of an indirect utility function which is twice continuously differentiable (see Kushner [1967, chap. 4] for more details). It is rigorously proven in Kushner (1967, chap. 4) that necessary conditions for the optimal controls are obtainable by solving

$$0 = \max_{[c(t), w_j(t)]} \{U[c(t), t] + \Delta J[W(t), t]\} \tag{8}$$

where $\Delta J[W(t), t]$ is defined by[3]

$$\lim_{\delta \to 0} \Big(E_t\{J[W(t + \delta), t + \delta] - J[W(t), t]\}/\delta - \Delta J[W(t), t] \Big) = 0.$$

This optimization problem can be solved sequentially as in

$$0 = \max_{\{c(t)\}} \{U[c(t), t] + \max_{\{w_j\}} \Delta J[W(t), t]\} \tag{9}$$

where $\max_{\{w_j\}} \Delta J[W(t), t]$ represents the trader's portfolio decision at time t given $c(t)$.

It can be shown that $J[W(t), t]$ is strictly increasing and strictly concave in $W(t)$.[4] Given this fact, ΔJ is analogous to an expected utility function which is strictly increasing and concave in $W(t + \delta)$ as $\delta \to 0$ or dW (see n. 3). Consequently, given that $\Delta J[W(t), t]$ is state independent (except wealth),[5] we can apply Ross's (1978, theorem 2, n. 10) mutual fund theorem to the portfolio decision in (9).

The condition given in the hypothesis of the theorem guarantees that $(r, dS_1/S_1, \ldots, dS_n/S_n)$ exhibits two-fund separation. Given that $(r, dS_1/S_1, \ldots, dS_n/S_n)$ exhibits two-fund separation, the risky mutual fund is the market portfolio. The proof of this statement is straightforward. Given homogeneous beliefs, assumption 6, everyone holds the same risky asset portfolio. In equilibrium, assumption 3, the only way this can happen is if this portfolio is the market portfolio.

3. For the stochastic process given in assumption 5,

$$[\Delta(J)] = \frac{\delta J}{\delta t} + \frac{\delta J}{\delta W}\left[\sum_{j=1}^{n} w_j(t)W(t)(\alpha_j - r) + rW(t) - c(t)\right]$$
$$+ \tfrac{1}{2}\frac{\delta^2 J}{\delta W^2}\left[\sum_{j=1}^{n}\sum_{i=1}^{n} w_i(t)w_j(t)W(t)^2\sigma_{ij}\right] + \sum_{j=1}^{n} \lambda_j E_t\{J[W(t)$$
$$+ w_j(t)W(t)\pi_j, t] - J[W(t), t]\}.$$

See Kushner (1967, p. 18) or Dreyfus (1965, p. 224).

4. The proof of this assertion is identical to a proof contained in Cox, Ingersoll, and Ross (1978) for an analogous proposition.

5. The necessary condition to apply Ross (1978) is that $\Delta(J)$ is state independent. If $\{\alpha_i, \sigma_i, \lambda_i, \pi_i\}$ is dependent on a state vector $l(t)$, then $\Delta\{J[W(t), t, l(t)]\}$ would depend on $l(t)$ (in general). Other sufficient conditions which give state independence are (i) logarithmic utility (see Kraus and Litzenberger 1975) and (ii) $\pi_i \equiv 0$ and $dl(t)$ independent of dZ_i (see Merton 1973*a*, Fama 1970).

Finally, an argument adapted from Ross (1978, p. 275) combined with the above completes the proof. Q.E.D.

The theorem gives a sufficient condition for the instantaneous CAPM even with asset prices having discontinuous sample paths. Using assumption 5a, the interpretation that the random component $(g_i d\eta_i - \lambda_i K_i dt + \pi_i dY_i)$ represents unsystematic risk is justified. The hypothesis guarantees that the risk is diversified away in the market portfolio.[6]

The above analysis is for a finite asset economy. Analogous results can be obtained using Ross's (1976) arbitrage pricing theory.[7] Merton (1976, p. 136) has shown the sufficiency of the analogous condition. However, under the APT a mutual fund theorem does not obtain because one does not need state-independent indirect utility functions.

Examining the sample path of the market portfolio can determine whether the hypothesis of the theorem is satisfied. This is the purpose of the next section of the paper.

III. Empirical Methodology and Results

This section examines the market portfolio's sample path to see if it contains a jump component. If no jump component is present, then this would be consistent with the hypothesis of the previous theorem and the satisfaction of an instantaneous CAPM. This section performs the following hypothesis test: H_0, jump risk diversifiable as in condition (6); H_A, jump risk nondiversifiable.

To perform this hypothesis test, we will examine the sample path of the market portfolio's return. To develop the testing methodology, note that under expression (4) the market portfolio's return dynamics are given by

$$\frac{dM}{M} = \sum_{j=1}^{n} m_j \alpha_j dt + \left(\sum_{j=1}^{n} m_j f_j\right) d\psi + \sum_{j=1}^{n} m_j (g_j d\eta_j - \lambda_j K_j dt + \pi_j dY_j), \tag{10}$$

where $M = \Sigma_{j=1}^{n} m_j S_j$. Under the null hypothesis, expression (10) reduces to

$$\frac{dM}{M} = \alpha dt + \sigma d\psi \tag{11}$$

6. Two other sufficient conditions which will yield an instantaneous CAPM are (i) $\pi_i \equiv 0$ or $\lambda_i \equiv 0$, for all i, and (ii) $\pi_i dY_i = f_i \pi dY$, for all i. Condition i is the condition given in Merton (1973*a*), and condition ii is where the jump component itself represents systematic movements across all stocks.

7. Under Ross's (1976) standard assumptions and our assumptions 1–7, with assumption 5 generalized to make $\{\alpha_i, \sigma_i, \lambda_i, K_i\}$ dependent on a state vector $I(t)$ it can be shown that if $\pi_i dY_i \equiv f_i \pi dY$, for all i, and $E(g_i d\eta_i)(g_j d\eta_j) = 0$, where $i \neq j$, then the APT holds and jump risk is compensated. If $\pi_i dY_i \neq f_i \pi dY$ and $E(g_i d\eta_i + dp_i)(g_j d\eta_j + dp_j) = 0, i \neq j$, where $dp_i = -\lambda_i K_i dt + \pi_i dYi$, then the APT holds and jump risk is diversifiable. The independence of the unsystematic risk is the key condition.

where $\alpha \equiv \Sigma_{j=1}^{n} m_j \alpha_j$ and $\sigma \equiv \Sigma_{j=1}^{n} m_j f_j$. Under the alternative hypothesis, expression (10) reduces to

$$\frac{dM}{M} = \alpha' dt + \sigma d\psi + dq, \tag{12}$$

where $dq \equiv \pi dY$ represents a Poisson process with parameter λ, $\pi \equiv$ jump amplitude with expected value equal to K, and $\alpha' \equiv \alpha - \lambda K$.

To complete the methodology, another hypothesis is added to (12), that is, (π) has a lognormal distribution with parameters (a, b^2). This assumption is added in order that the maximum likelihood estimation technique as developed in Rosenfeld (1982) can be used to estimate the parameters of equations (11) and (12).

For ease of reference, we repeat the hypothesis to be tested:

H_0, jump risk diversifiable,

$$\frac{dM}{M} = \alpha dt + \sigma d\psi; \tag{13}$$

H_A, jump risk not diversifiable,

$$\frac{dM}{M} = \alpha' dt + \sigma d\psi + dq; \tag{14}$$

and (π) is distributed lognormal (a, b^2).

The likelihood function corresponding to equation (15) is given by

$$L_u = \frac{e^{-N\lambda h}}{2\pi^{N/2}} \prod_{i=1}^{N} \sum_{j=0}^{\infty} \left\{ \frac{1}{(\sigma^2 h + b^2 j)^{1/2}} \exp \left[\frac{-(\ln \frac{M_i}{M_{i-1}} - \mu h - \theta j)^2}{2(\sigma^2 h + b^2 j)} \right] \frac{(\lambda h)^j}{j!} \right\},$$

where N is the number of observations, h is the increment of time between observations, $\mu = \alpha - \sigma^2/2$, and $\theta = a - b^2/2$. Alternatively, the likelihood function corresponding to equation (13) is given by

$$L_c = \prod_{i=1}^{N} \frac{1}{(2\pi\sigma^2 h)^{1/2}} \exp \left\{ \frac{-(\ln \frac{M_i}{M_{i-1}} - \mu h)^2}{2\sigma^2 h} \right\}.$$

To formally test the null versus the alternative hypothesis, a likelihood ratio test can be used: $\Lambda = -2(\ln L_c - \ln L_u)$, where L_c represents the likelihood value for the constrained density function (i.e., the null hypothesis, eq. [13]) and L_u represents the likelihood function for the unconstrained density function (i.e., the alternative hypothesis, eq.

[14]). The likelihood ratio statistic, Λ, is distributed asymptotically χ^2 with 3 *df*.[8]

The empirical tests were performed on two market indices. The first index consists of daily observations for a value weighted portfolio of all stocks on the New Year Stock Exchange and the American Stock Exchange from July 1962 to December 1978. Both of these market indices were taken from the CRSP tapes and include the reinvestment of all dividends.

Table 1 presents estimates of the parameters of the diffusion-only process for different observation intervals and time periods. The asymptotic standard errors of the estimates are in parentheses below the point estimates. The results suggest that the ex post average return and volatility of the market are not constant over time. The overall standard deviation of return on the market as measured by the New York Stock Exchange value-weighted index over a 50-year period is on the order of 20% ($\sqrt{.04}$), while over the last 20 years the volatility has decreased to 14% ($\sqrt{.019}$).

Some evidence relating to whether the market portfolio contains a jump component is given by the empirical distributions in table 2. For the daily market index, there are 2 out of 4,133 observations that exhibit a daily return of greater than 5%. This is a movement of approximately 5 S.D., which makes these two observations prime candidates for jumps. For the monthly observation NYSE index, there are 14 out of 633 observations which have larger than 15% movements (1.2 S.D.) in a given month. This in itself does not indicate a significant jump component for this market index.

In table 3 estimates are given for the combined diffusion and jump process. A simple *t*-test of the jump parameter indicates that for some of the daily sample periods a statistically significant jump component exists. This is confirmed by the likelihood ratio test. All of the daily sample periods rejected the null hypothesis of a continuous sample path process at a 99% significance level.

It is interesting to observe that when daily data are aggregated to either weekly or monthly data, the jump process appears to disappear. It may be that measurement errors associated with daily data induce

8. The significance levels for this distribution are given below:

Probability of Rejecting the Null Hypothesis	χ^2 Value
.5	2.37
.75	4.11
.9	6.25
.95	7.81
.99	11.35

TABLE 1 Diffusion Process Market Indices

Index	No. of Observations	$\hat{\mu}$	$\hat{\sigma}^2$	$\ln L_u$
MKT	4,133	.0009 (.039)	.019 (.0004)	14324.6
MKT1	800	.226 (.06)	.0087 (.0004)	3082.9
MKT2	800	.166 (.07)	.012 (.0006)	2962.9
MKT3	800	.02 (.09)	.019 (.001)	2758.8
MKT4	800	−.056 (.12)	.035 (.0017)	2528.9
MKT5	933	.003 (.145)	.083 (.019)	3275.1
MKTM	198	.076 (.035)	.02 (.002)	347.6
MKTW	860	.076 (.036)	.02 (.001)	2109.4
MKTW1	430	.084 (.047)	.018 (.001)	1104.5
MKTW2	430	.068 (.057)	.027 (.002)	1014.5
NYSE	633	.078 (.028)	.04 (.002)	896.6
NYSE1	317	.068 (.049)	.064 (.005)	380.3
NYSE2	316	.088 (.027)	.019 (.001)	571.9

NOTE.—Value-weighted indices including all stocks on the New York Stock Exchange and the American Stock Exchange: MKT = daily observations from July 1962 to December 1978; MKT1 = daily observations from July 1962 to August 1965; MKT2 = daily observations from September 1965 to October 1968; MKT3 = daily observations from November 1968 to December 1971; MKT4 = daily observations from January 1972 to March 1975; MKT5 = daily observations from April 1975 to December 1978; MKTM = monthly observations from July 1962 to December 1978; MKTW = weekly observations from July 1962 to December 1978; MKTW1 = weekly observations from July 1962 to September 1970; MKTW2 = weekly observations from October 1970 to December 1978.

Value-weighted indices including all stocks on the New York Stock Exchange: NYSE = monthly observations from January 1927 to December 1978; NYSE1 = monthly observations from January 1927 to December 1952; NYSE2 = monthly observations from January 1953 to December 1978.

Standard errors are in parentheses below the point estimates.

"jumps" in the market return. In daily data the failure to adjust for weekends and holidays could lead to a false determination of the existence of a jump process. The variance of return associated with these observations (i.e., weekends and holidays) is greater than average, thereby causing movements on these days to appear like jumps. Since we know the length of time between successive observations, h_i, we could simply normalize the actual return data by dividing each observation by h_i. However, empirical evidence concerning the behavior of market returns when the stock exchange is closed (see Granger and Morgenstern 1970) indicates that during this period the volatility is decreased. As such, only a partial adjustment is required for the num-

TABLE 2 Market Indices Empirical Distributions

No. of Observations	Range (%)	Number	Percentage
Market index: value weighted including all stocks on the New York Stock Exchange and the American Stock Exchange (maximum = 9.3%, minimum = −3.5%)			
4,133 daily observations: July 1962–December 1978	>5	2	0
	4 to 5	3	.1
	3 to 4	7	.2
	2 to 3	38	.9
	1 to 2	221	5.3
	0 to 1	1,963	47.5
	−1 to 0	1,583	38.3
	−2 to −1	279	6.8
	−3 to −2	32	.8
	−4 to −3	4	.1
	−5 to −4	0	0
	< −5	0	0
Value weighted including all stocks on the New York Stock Exchange (maximum = 38.5%, minimum = −29.1%)			
633 monthly observations: January 1927–December 1978	>15	7	1.1
	12 to 15	8	1.3
	9 to 12	8	1.3
	6 to 9	46	7.3
	3 to 6	139	22
	0 to 3	181	28.5
	−3 to 0	125	19.7
	−6 to −3	65	10.3
	−9 to −6	25	3.9
	−12 to −9	14	2.2
	−15 to −12	8	1.3
	<−15	7	1.1

ber of days during which the exchange is closed. We have arbitrarily chosen a square root adjustment (i.e., from Friday to Monday is 3 days, so $h_i = \sqrt{3/360}$).[9]

In table 4 new estimates are presented for the daily market returns corrected for the measurement errors. The results indicate that there are measurement errors and they do tend to induce jumps. Even after the correction procedure, however, there still appears to be a significant jump component in all of the daily subperiods, except (MKT3).

Since our correction procedure is dependent on the type of adjustment used (i.e., square root) an alternative test would be to eliminate all observations which span weekends and/or holidays. The results for this test are presented in table 5. Again, a significant jump component still persists in the daily subperiods. Although there appears to be measurement error associated with weekends and holidays, after ad-

9. A square root adjustment seemed to fit the empirical results in Granger and Morgenstern (1970). For a formal treatment of the problem, the adjustment factor could be estimated along with the other parameters of the stochastic process.

TABLE 3 Combined Diffusion and Jump Process

Index	No. of Observations	Market Indices $\hat{\alpha}$	$\hat{\sigma}$	$\hat{\lambda}$	$\hat{a}$	$\hat{b}$	$\ln L_u$	Λ
MKT1	800	.34	.06	78.4	−.0014	.0078	3,172.3	178.8
		(.05)	(.003)	(20.5)	(.0008)	(.0008)		
MKT2	800	.24	.08	262.9	−.0005	.0033	2,976.1	26.4
		(.18)	(.02)	(509.3)	(.0007)	(.0021)		
MKT3	800	−.046	.10	78.5	.001	.0104	2,794.5	71.4
		(.10)	(.007)	(36.1)	(.0012)	(.0017)		
MKT4	800	.41	.084	224.2	−.0011	.0034	2,980.0	902.2
		(.31)	(.018)	(441.3)	(.0015)	(.002)		
MKT5	933	.13	.13	3.0	.036	.03	3,300.1	50.0
		(.07)	(.003)	(4.4)	(.07)	(.05)		
MKTM	198	.10	.13	.6	−.01	.03	347.6	0.0
		(.05)	(.01)	(.4)	(.04)	(.04)		
MKTW	860	.15	.16	1.8	.027	.03	2,112.2	5.6
		(.05)	(.004)	(.9)	(.012)	(.007)		
MKTW1	430	.13	.13	1.7	.03	.03	1,115.1	21.2
		(.06)	(.005)	(1.2)	(.01)	(.006)		
MKTW2	430	.07	.17	3.8	−.0002	.03	1,014.3	0.0
		(.09)	(.009)	(1.3)	(.01)	(.009)		
NYSE	633	.10	.19	2.15	.001	.016	898.6	4.0
		(.075)	(.01)	(1.25)	(.005)	(.006)		
NYSE1	317	.11	.15	3.4	−.001	.04	381.8	3.0
		(.11)	(.03)	(3.5)	(.004)	(.01)		
NYSE2	316	.13	.14	.007	.11	.01	575.4	7.0
		(.05)	(.01)	(.88)	(.23)	(.02)		

Note.—Standard errors are in parentheses below the point estimates.

TABLE 4 **Market Indices: Daily Data**
(Adjusted for Weekends and Holidays)

	Diffusion Process			Combined Diffusion and Jump Process						
Index	$\hat{\mu}$	$\hat{\sigma}^2$	$\ln L_c$	$\hat{\alpha}$	$\hat{\sigma}$	$\hat{\lambda}$	$\hat{a}$	$\hat{b}$	$\ln L_u$	Λ
MKT1	.23	.007	3,174.1	.32	.05	87.0	−.001	.006	3,199.6	51.0
	(.05)	(.0004)		(.18)	(.03)	(14.2)	(.01)	(.02)		
MKT2	.19	.009	3,014.2	.31	.07	242.3	−.0002	.007	3,057.6	86.8
	(.06)	(.0004)		(.16)	(.02)	(148.6)	(.001)	(.001)		
MKT3	.08	.016	2,856.3	−.01	.09	62.1	.001	.008	2,863.8	15.0
	(.08)	(.0008)		(.09)	(.007)	(46.4)	(.001)	(.001)		
MKT4	.009	.029	2,863.4	.27	.14	122.3	.001	.003	2,997.6	268.4
	(.11)	(.0014)		(.38)	(.02)	(59.3)	(.001)	(.002)		
MKT5	.14	.017	3,294.9	.11	.13	.4	.05	.03	3,315.8	41.8
	(.08)	(.0008)		(.003)	(.003)	(1.6)	(.09)	(.05)		

NOTE.—Standard errors are in parentheses below the point estimates.

TABLE 5 **Market Indices: Daily Data**
(Weekends and Holiday Observations Eliminated)

Index	No. of Observations	Diffusion Process			Combined Diffusion and Jump Process						
		$\hat{\mu}$	$\hat{\sigma}^2$	$\ln L_c$	$\hat{\alpha}$	$\hat{\sigma}$	$\hat{\lambda}$	$\hat{a}$	$\hat{b}$	$\ln L_u$	Λ
MKT1	622	.29	.0079	2,430.2	.48	.05	68.7	−.002	.007	2,473.1	85.8
		(.065)	(.0005)		(.06)	(.004)	(31.6)	(.001)	(.001)		
MKT2	592	.32	.0099	2,243.2	.34	.05	229.8	−.0004	.004	2,256.9	27.2
		(.075)	(.0006)		(.11)	(.01)	(176.3)	(.0002)	(.001)		
MKT3	626	.196	.018	2,190.5	.29	.06	230.7	−.001	.006	2,201.7	22.4
		(.097)	(.001)		(.13)	(.02)	(119.2)	(.001)	(.001)		
MKT4	621	.111	.033	1,980.9	.42	.12	199.3	−.001	.003	2,008.9	56.0
		(.133)	(.022)		(.36)	(.03)	(221.6)	(.002)	(.003)		
MKT5	733	.165	.02	2,517.7	.11	.13	.8	.05	.03	2,538.7	42.0
		(.096)	(.001)		(.09)	(.005)	(6.1)	(.28)	(.04)		

NOTE.—Standard errors are in parentheses below the point estimates.

justment the data are still inconsistent with the null hypothesis. There appears to be a jump process operating, but the magnitudes of the jumps are very small. In fact, for (MKT2, MKT4, MKT5) the estimated $\hat{\lambda}$ is insignificant from zero.

In summary, after adjusting for weekends and holidays, the null hypothesis is rejected for daily data. However, the magnitudes of the jumps are small. In the weekly and monthly data, the null hypothesis cannot be rejected. A plausible reason for that is that with the weekly and monthly observation intervals, the inclusion of weekends and holidays within the interval tends to cover up (hide) the small jump components. With daily observations, when one includes weekend and holiday returns just the reverse occurs (i.e., jumps are accentuated). This is consistent with the reduction of the likelihood ratios in tables 4 and 5 after adjusting for weekends and holidays.

IV. Conclusion

This paper both develops and tests sufficient conditions for an instantaneous CAPM when stock returns follow a jump-diffusion process. Based on daily returns, our conclusions are that the market portfolio contains a jump component although its magnitude is small. When measured over larger intervals in time (weekly or monthly intervals), the presence of weekends and holidays tends to cover up the small jump component. The economic implication is that jump risk is not diversifiable and an instantaneous CAPM (as given by the theorem in Sec. II) will not hold.

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