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Intradaily dynamic portfolio selection

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ABSTRACT

A portfolio selection model which allocates a portfolio of currencies by maximizing the expected return subject to Value-at-Risk (VaR) constraint is designed and implemented. Based on an econometric implementation using intradaily data, the optimal portfolio allocation is forecasted at regular time intervals. For the estimation of the conditional variance from which the VaR is computed, univariate and multivariate GARCH models are used. Model evaluation is done using two economic criteria and two statistical tests. The result for each model is given by the best forecasted intradaily investment recommendations in terms of the optimal weights of the currencies in the risky portfolio. The results show that estimating the VaR from multivariate GARCH models improves the results of the forecasted optimal portfolio allocation, compared to using a univariate model.

1. Introduction

We design and implement a financial model for forecasting optimal portfolio allocations of currencies using intradaily data. The optimal allocation maximizes the expected portfolio return subject to VaR constraint. In the econometric implementation of the model, we deal with portfolios made of two or three currencies, among the euro (EUR), the Great Britain pound (GBP), and the Japanese yen (JPY), with the numeraire being the US dollar (USD). Our model is set up for foreign exchange dealers who re-balance their portfolio of currencies at regular time intervals during each trading day and must satisfy a daily VaR constraint.

We use several econometric models to compute the optimal portfolios. Each model is estimated using historical data up to a certain date. Each estimated model then serves to generate, for the next time interval, an out-of-sample forecast of the expected return and a quantile of the future return distribution, which are used as inputs to compute the optimal allocation of risky currencies. The optimization step also determines the optimal amount of numeraire that dealers are allowed to borrow or lend. The whole procedure is carried out sequentially by adding one observation at a time to the estimation sample and generating the forecasted investment recommendations for the next time interval.

Our theoretical portfolio allocation model is based on the model of Campbell et al. (2001) who propose allocating financial assets by maximizing the expected return, subject to the constraint that the expected maximum loss should meet the VaR constraint. We consider a dynamic model instead of a static one and investigate an intradaily portfolio rebalancing instead of interdaily rebalancing. We take into consideration, in this case, market microstructural issues such as intradaily seasonality (i.e. market opening and closing, lunch time, …etc.); we implement univariate and multivariate models to estimate portfolio returns and VaR; we predict and evaluate portfolio weights in an out-of-sample setting; finally, we check the robustness and statistical significance of our results using economic criteria and statistical tests.

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In this way, we use both univariate and multivariate GARCH and non-GARCH models and compare the performance of the different models on the basis of two economic criteria (the generated wealth and a performance ratio) and two statistical tests (the failure rate and the dynamic quantile tests). Our results indicate clearly that using multivariate GARCH models improves the results of the optimal portfolio allocation in terms of the evaluation criteria.

Choosing foreign exchange markets for empirical evidence has a double advantage. Firstly, we avoid making assumptions (sometimes not supported by financial theory) to simplify computations. For example intradaily rebalancing in stock markets would not be practically meaningful, since there are significant transaction costs and fees involved, while in the case of foreign exchange trading, there are no transaction costs other than the bid–ask spread. Secondly, foreign exchange data allow us to be closer to real trading, to the extent that foreign exchange dealers rebalance, intradaily, their currency portfolio and realize their profits or losses at the end of each business day.

Many studies have investigated dynamic portfolio selection under the variance and value-at-risk (VaR) constraints. All of them focus on stock and bond portfolios and consider daily frequency data.

Pojarliev and Polasek (2003) analyze the performance of stock portfolio strategies in a mean–variance framework. They implement an asymmetric GARCH model and the multivariate BEKK model to forecast the portfolio variance. Based on the Sharpe ratio, they find that the multivariate GARCH model outperforms the univariate one. Alexander and Baptista (2004) compare portfolio selection implications arising from imposing a VaR constraint on their model versus the imposition of a conditional value-at-risk (CVaR) constraint. They show that, for a given confidence level, a CVaR constraint is tighter than a VaR constraint if the CVaR and VaR bounds coincide; but in the absence of a risk-free security, CVaR has a perverse effect in that it is more likely to force highly risk-averse agents to select portfolios with larger risk. De Goeij and Marquering (2004) analyzes bond and stock market interactions by modeling conditional covariances using diagonal and asymmetric VECH models. They show that daily returns on the S&P 500 and NASDAQ indexes exhibit significant leverage effects and find substantial gains due to asymmetric volatility timing. Giannopoulos et al. (2005) show that VaR computation based on the empirical variance underestimates stock portfolio risk compared to a univariate GARCH model. Zhang and Huang (2006) proposes a new approach that combines Markov and max-stable processes to calculate the VaR and evaluate portfolio allocations under VaR constraints. A stochastic process $\{Y_t\}_{t\in T}$ is called a max-stable process if the following property holds: if $\{Y_t^{(i)}\}_{t\in T}$, $i = 1, 2, \ldots, r$, are independent copies of the process, then the process $\{\max_{i\leq r} Y_t^i\}_{t\in T}$ has the same distribution as $\{rY_t^{(1)}\}_{t\in T}$ ((De Haan, 1984)). They find that VaR based on their approach outperforms the empirical variance–covariance approach and that the latter underestimates the risks during recession and expansion time.

Specht and Winker (2008) use a Principal Component GARCH (PC-GARCH) model to compute the conditional variance–covariance matrix and estimate the VaR. Using the latter as a constraint and the portfolio expected return as a performance measure, they show that PC-GARCH outperforms the empirical variance model. To estimate the variance–covariance matrix, they implement univariate and multivariate GARCH models.

In Section 2, we present the portfolio allocation model. In Section 3, we describe the econometric models we use for the prediction of the VaR. In Section 4, we present the empirical illustration using intradaily exchange rate data. In the last section, we conclude.

2. Portfolio allocation model

Although VaR is a very popular measure of risk, it has been criticized because it does not satisfy one of the four properties for coherent risk measure, namely subadditivity (Artzner et al., 1999; Rockafellar and Uryasev, 2000; Szegö, 2002). The rest of the coherent risk measure properties are: positive homogeneity, monotonicity and transitional invariance. However, VaR becomes subadditive and can be considered as a coherent risk measure, if used in the case of elliptic joint distributions, like the normal and Student distributions with finite variances (Embrechts et al., 1999; Szegö, 2002). Our empirical study considers only Normal and Student-*t* distributions. In what follows, we take the VaR as industry-standard and we implement the model of Campbell et al. (2001) including some adjustments (such as model dynamics, market microstructural issues, univariate and multivariate VaR estimations) to find the optimal intradaily portfolio allocation of currencies that maximizes the expected portfolio return subject to a VaR constraint. We make the following assumptions:

- Dealers hold an initial wealth involving only the numeraire.
- Dealers re-balance their currency portfolio regularly (e.g. every thirty minutes) during each trading day. They make capital gains or losses, due to exchange rate fluctuations, and interest gains or losses from their wealth lent or borrowed at the end of the day. At the end of the day, dealers have to close their risky positions in order to meet their daily inventory control guideline causing the amount of the overnight currency interest rate to be roughly null. We only consider the gains or losses coming from the overnight interest rate related to numeraire and the exchange rate fluctuations. If there are no currency positions, dealers have to lend their total wealth involving only the numeraire.
- There are no transaction costs except the bid–ask spread. This assumption is consistent with the stylized fact found in FX markets, that dealers pay the ask price when they buy and charge the bid price when they sell.
- There is no interest rate spread; i.e. the daily interest rate at which dealers can borrow or lend is the same. In our context, this assumption is acceptable since the difference between the lending and borrowing overnight interest rates is very small. In addition, the amount of numeraire to borrow or lend at the end of each day equals the cumulative counter-value of the long and short positions taken by the dealer along the day, each time he re-balances his portfolio. The daily amount

of interest received or paid is equivalent to the cumulative intradaily one. Indeed, this allows the computational of the interest rate amount just before each re-balancing, and the estimation of the net total daily amount.
- The interest rate remains unchanged over the out-of-sample period of 30 days.
- The desired amount of numeraire that dealers may want to borrow should be obtained quickly because of the high speed of intradaily operations. The borrowing can take the form of a "count in advance" that involves no trading costs. If a dealer represents a bank, the borrowing and lending inside the same institution is, most of the time, granted.

Let $n+1$ be the number of available currencies for trading. Taking the $(n+1)$-th currency as numeraire, in our case the USD, let $p_{i,t}$, with $i = 1, \ldots, n$, be the exchange rate between currencies i and $n+1$ expressed in units of currency $n+1$ per unit of currency i (e.g. 1.25 USD per 1 EUR). Define the return of currency i at time t as $r_{i,t-1} = \log(p_{i,t}) - \log(p_{i,t-1})$, for $i = 1, \ldots, n$.

Let W_{t-1} be the dealer's wealth, in USD, at time t-1. Define $\Omega_{t-1} \equiv \{w_{t-1} \in R^n : \sum_{i=1}^n w_{i,t-1} = 1\}$ as the set of portfolio weights at time $t-1$. Note that this formulation allows for short-selling of some of the currencies in the portfolio and assumes that the income from these short-sales is invested in the other currencies of the portfolio. Then, $x_{i,t-1} = w_{i,t-1}W_{t-1}/p_{i,t-1}$ represents the amount of currency i held at time $t-1$. This implies:

$$x_{i,t-1}p_{i,t-1} = w_{i,t-1}W_{t-1}, \tag{2.1}$$

and

$$W_{t-1} = \sum_{i=1}^{n} x_{i,t-1}\, p_{i,t-1}. \tag{2.2}$$

The wealth at time t can be expressed as:

$$W_t = W_{t-1}\left(1 + \sum_{i=1}^{n} w_{i,t-1}\, r_{i,t}\right). \tag{2.3}$$

We allow the dealers to borrow or to lend USD according to their degree of risk aversion. If dealers are less risk averse, they would like to borrow in order to invest this money in currencies that allow them to maximize their expected returns. This borrowing can be seen as a leverage to obtain a higher return.

Denote by b_{t-1} the amount of USD that a dealer can borrow ($b_{t-1} > 0$) or lend ($b_{t-1} < 0$) at the risk-free interest rate r_f. With borrowing and lending, Eq. (2.3) becomes

$$W_t = (W_{t-1} + b_{t-1})\left(1 + \sum_{i=1}^{n} w_{i,t-1} r_{i,t}\right) - b_{t-1}(1 + r_f). \tag{2.4}$$

Dealers wish to maximize their expected wealth since future returns are not known. Accordingly, Eq. (2.4) in terms of conditional expectations is given by

$$E_{t-1}[W_t(w_{t-1})] = (W_{t-1} + b_{t-1})(1 + E_{t-1}[r_t(w_{t-1})]) - b_{t-1}(1 + r_f), \tag{2.5}$$

where $E_{t-1}[r_t(w_{t-1})]$ is the expected portfolio return at the end of the rebalancing period. The expectation operator E_{t-1} is conditional on all the information available at time $t-1$. In order to maximize this objective function, dealers face two constraints: the budget constraint and the risk constraint given in terms of the VaR. The budget constraint is given by

$$W_{t-1} + b_{t-1} = \sum_{i=1}^{n} x_{i,t-1}p_{i,t-1} = x'_{t-1}p_{t-1}. \tag{2.6}$$

The VaR constraint for a given intradaily time interval and probability of occurrence α is

$$P_{t-1}[W_t(w_{t-1}) \le W_{t-1} - VaR^*] \le \alpha, \tag{2.7}$$

where P_{t-1} is the probability given all the information available at time $t-1$, and VaR^* is the dealer's desired VaR level.

The intradaily portfolio optimization problem is solved by the maximization of the expected return, Eq. (2.5), subject to the budget constraint, Eq. (2.6), the VaR-constraint, Eq. (2.7), and the simplex constraint ($\sum_{i=1}^n w_{i,t-1} = 1$). The objective is to determine the weights that maximize the expected return subject to those constraints, i.e.

$$w^*_{t-1} \equiv \arg\max_{w_{t-1}} (W_{t-1} + b_{t-1})(1 + E_{t-1}[r_t(w_{t-1})]) - b_{t-1}(1 + r_f). \tag{2.8}$$

Substituting the budget constraint (2.6) in the objective function (2.5) yields

$$E_{t-1}[W_t(w_{t-1})] = x'_{t-1}p_{t-1}(E_{t-1}[r_t(w_{t-1})] - r_f) + W_{t-1}(1 + r_f). \tag{2.9}$$

Eq. (2.9) shows that a risk-averse dealer is ready to invest a fraction of his wealth in foreign currencies if the expected return of the portfolio is bigger than the risk free rate.

Substituting (2.9) (before taking the expectation) in (2.7) gives

$$P_{t-1}[x'_{t-1}p_{t-1}(r_t(w_{t-1}) - r_f) + W_{t-1}(1 + r_f) \le W_{t-1} - VaR^*] \le \alpha, \tag{2.10}$$

such that

$$P_{t-1}\left[r_t(w_{t-1}) \le r_f - \frac{VaR^* + W_{t-1}r_f}{x'_{t-1}p_{t-1}}\right] \le \alpha \tag{2.11}$$

defines the quantile $q_t(w_{t-1}, \alpha)$ of the distribution of the portfolio return at the confidence level α. Using this result, the investment value can be written as

$$x'_{t-1}p_{t-1} = \frac{VaR^* + W_{t-1}r_f}{r_f - q_t(w_{t-1}, \alpha)}. \tag{2.12}$$

Finally, substituting (2.12) in (2.9) and dividing by the initial wealth W_{t-1} we obtain

$$\frac{E_{t-1}[W_t(w_{t-1})]}{W_{t-1}} = \frac{VaR^* + W_{t-1}r_f}{W_{t-1}r_f - W_{t-1}q_t(w_{t-1}, \alpha)}(E_{t-1}[r_t(w_{t-1})] - r_f) + (1 + r_f). \tag{2.13}$$

Thus, the optimal set of weights is given by

$$w^*_{t-1} \equiv \arg\max_{w_{t-1}} \frac{E_{t-1}r_t(w_{t-1}) - r_f}{W_{t-1}r_f - W_{t-1}q_t(w_{t-1}, \alpha)} = \arg\max_{w_{t-1}}(PR(w_{t-1})). \tag{2.14}$$

The computation of the optimal weights is done numerically by performing a grid search over a range of values of w_{t-1}. In our applications, the dimension of the latter is two or three (the number of risky currencies), but since the sum of the weights must be equal to one, the dimension of the problem is reduced to one or two. For two currencies, we make a grid search over the interval [−100%, +100%] (since short selling is possible) for the first element of w_{t-1} and the second element is set to 100 minus the first. For a dimension of two, the interval for each of the first two elements of w_{t-1} is [−200%, 100%] and the last element is obtained as 100 minus the sum of the first two. Note that W_{t-1} does not affect the optimal portfolio weights since it is a scale constant in the maximization. $PR(w_{t-1})$ can be interpreted as the ratio of expected risk premium related to the currencies portfolio to the risk. The denominator could be considered as a measure for regret, since it measures the potential opportunity loss in investing in portfolio of currencies. Indeed, $PR(w_{t-1})$ is a portfolio performance measure like the Sharpe (1964) ratio. The latter is considered as a performance measure within the mean–variance framework and is given by $\frac{E_{t-1}r_t(w_{t-1})-r_f}{\sigma_t(w_{t-1})}$.

The dealer's initial wealth and desired VaR^* do not affect the maximization of $PR(w_{t-1})$. Dealers first allocate the currencies and then the amount of borrowing or lending. Thus the well-known two fund separation theorem (see for example Huang and Litzenberger (1988)) holds, as in the mean–variance framework. The amount to borrow or to lend reflects by how much the VaR of the portfolio, $W_{t-1}q_t(w_{t-1}, \alpha)$, differs from the desired VaR^*. The amount of numeraire that a given dealer borrows or lends is obtained by substituting (2.6) in (2.12), which gives

$$b^*_{t-1} = \frac{VaR^* + W_{t-1}q_t(w^*_{t-1}, \alpha)}{r_f - q_t(w^*_{t-1}, \alpha)}. \tag{2.15}$$

We use this model to compute intradaily portfolio allocations for trading in FX markets. In this context, the time interval during which the optimization is conducted is supposed to be short. This fact introduces some high-frequency data aspects that must be considered in the econometric implementation to estimate the optimal portfolio allocations. An important aspect to consider is the seasonal component present in high frequency data. In the next section, we explain, in detail, the econometric implementation of the model and how we deal with the high-frequency features of the data.

3. Econometric implementation

We want to compute w^*_{t-1} and b^*_{t-1} as defined in Eqs. (2.14) and (2.15), respectively. To implement this, we need to estimate $E_{t-1}[r_t(w_{t-1})]$ and $q_t(w^*_{t-1}, \alpha)$ from historical data (i.e. until date $t-1$ inclusive), using an econometric model of the portfolio return distribution. We can do this sequentially for a sequence of periods and evaluate the ex-post performance of the investments recommended by the econometric procedure.

Our methodology for the estimation of the optimal portfolio allocation and its evaluation comprise five steps that we describe below.

3.1 Adjustment of each currency return for intradaily seasonality
We start by estimating the intradaily component, $\phi_i(t)$, which characterizes the second moment of the returns of currency i at time t of the day. Following Andersen and Bollerslev (1997) and Bauwens et al. (2005), this intradaily seasonal (or diurnal) component is defined as the expected volatility conditioned on time-of-day, where the expectation is computed by averaging the squared observed returns over the cross-sectional time intervals for each day of the week.

In order to get rid of the diurnal pattern of the volatility of intradaily returns, we adjust the returns by dividing them by the square root of $\phi_i(t)$ for $i = 1, \ldots, n$, to get "deseasonalized" returns. If $r_{i,t}$ is the observed return of currency i at time t, and $R_{i,t}$ is the deseasonalized one, then $R_{i,t} = r_{i,t}/\sqrt{\phi_i(t)}$. Actually, the function $\phi_i(t)$ is different for each day of the week (see Section 4.1).

3.2 Specification and estimation of an econometric model of portfolio returns

For the specification of an econometric model of portfolio returns, we distinguish two cases: the univariate and multivariate cases. In the univariate case, we model the deseasonalized portfolio returns $R_{p,t}$ as follows:

$$R_{p,t} = \mu_{p,t} + \epsilon_t, \tag{3.1}$$

where $\mu_{p,t}$ is the conditional mean and ϵ_t an error term.

In the multivariate setting, we replace $R_{p,t}$ by a $n \times 1$ vector R_t which contains the returns $R_{i,t}$, $i = 1, \ldots, n$, of the n foreign currencies at time t:

$$R_t = \mu_t + \epsilon_t, \tag{3.2}$$

where μ_t and ϵ_t are $n \times 1$ vectors.

3.2 (a) Specification of the conditional mean

We specify the conditional mean of $R_{i,t}$, with $i = p$ in the univariate case, and $i = 1, \ldots, n$ in the multivariate case, as the AR(1) process

$$\mu_{i,t} = \mu_i + \rho_i R_{i,t-1}, \tag{3.3}$$

or as a slowly changing parameter that is estimated by the mean of the observations until time $t - 1$:

$$\mu_{i,t} = E_{t-1}(R_{i,t}), \qquad \hat{\mu}_{i,t} = \frac{1}{t}\sum_{s=0}^{t-1} R_{i,s}. \tag{3.4}$$

3.2 (b) Specification of the conditional variance

In the univariate case, Eq. (3.1), the error term ϵ_t is decomposed as $\sigma_{p,t} z_t$ where z_t is an IID process with zero mean and unit variance. In this case, given a portfolio allocation, we compute the deseasonalized portfolio returns to estimate the conditional variance $\sigma^2_{p,t}$ using two univariate specifications:

(a) A "non-parametric" model in which we assume that $\sigma^2_{p,t}$ is slowly evolving in time and is estimated at time t by the empirical variance of the data until $t - 1$.

$$\sigma^2_{p,t} = Var_{t-1}(R_{p,t}), \qquad \hat{\sigma}^2_{p,t} = \frac{1}{t}\sum_{s=0}^{t-1}(R_{p,s} - \hat{\mu}_{p,t})^2. \tag{3.5}$$

(b) The GARCH(1,1) model of Bollerslev (1986), written

$$\sigma^2_{p,t} = \omega_p + \delta_p \epsilon^2_{t-1} + \beta \sigma^2_{p,t-1}. \tag{3.6}$$

In the multivariate case, Eq. (3.2), the error term ϵ_t equals $\Sigma_t^{1/2} z_t$, where $\Sigma_t^{1/2}$ is, for example, the Cholesky factorization of the $n \times n$ conditional variance–covariance matrix Σ_t, and where the $n \times 1$ vector z_t is an IID process with mean zero and variance I_n (the identity matrix of order n). For this case, we use three multivariate GARCH models (see Bauwens et al. (2006) for a detailed presentation of multivariate GARCH models). We consider also a "non-parametric" model in which we assume that Σ_t is slowly evolving in time:

(c) The dynamic conditional correlation (DCC) model of Tse and Tsui (2002), defined by

$$\Sigma_t = D_t \Lambda_t D_t. \tag{3.7}$$

The $n \times n$ matrix D_t is a diagonal matrix containing the conditional variances $\sigma^2_{i,t}$, for $i = 1, 2, \ldots, n$, each specified as

$$\sigma^2_{i,t} = \omega_i + \delta_i \epsilon^2_{i,t-1} + \beta_i \sigma^2_{i,t-1}, \tag{3.8}$$

i.e. a univariate GARCH(1,1) equation. The $n \times n$ matrix Λ_t is the conditional correlation matrix, defined by

$$\Lambda_t = (1 - \theta_1 - \theta_2)\Lambda + \theta_1 \Psi_{t-1} + \theta_2 \Lambda_{t-1}, \tag{3.9}$$

where Λ is a constant correlation matrix, θ_1 and θ_2 are non-negative parameters which should be satisfying $\theta_1 + \theta_1 < 1$, and Ψ_{t-1} is the $n \times n$ correlation matrix of ϵ_τ for $\tau = t - M, t - M + 1, \ldots, t - 1$. Its i, j-th element is given by

$$\Psi_{ij,t-1} = \frac{\sum_{m=1}^{M} u_{i,t-m} u_{j,t-m}}{\sqrt{\left(\sum_{m=1}^{M} u^2_{i,t-m}\right)\left(\sum_{m=1}^{M} u^2_{j,t-m}\right)}}, \tag{3.10}$$

where $u_{i,t} = \epsilon_{it}/\sigma_{i,t}$. The scalar M must be greater than or equal to n to ensure that Ψ_t is semi-positive-definite. Notice that the right-hand side of (3.9) is such that the diagonal elements of Λ_t are equal to one for all i and t (assuming that Λ_0 is a correlation matrix).

(d) The constant conditional correlation (CCC) model of Bollerslev (1990), is a particular case of the DCC where $\theta_1 = \theta_2 = 0$ and

$$\Lambda_t = \Lambda. \tag{3.11}$$

(e) The BEKK(1,1,1) model of Engle and Kroner (1995), where the conditional variance–covariance matrix is defined by

$$\Sigma_t = C'C + A'\epsilon_{t-1}\epsilon'_{t-1}A + G'\Sigma_{t-1}G, \tag{3.12}$$

where A and G are $n \times n$ matrices, and C is upper triangular.

(f) The slowly evolving in time covariance (COV), where Σ_t is estimated at time t by the empirical covariance of the data until $t-1$, and for $i, j = 1, \dots, n$:

$$\Sigma_t = COV_{t-1}(R_t), \qquad \hat{\Sigma}_t = \frac{1}{t}\sum_{s=0}^{t-1}(R_s - \hat{\mu}_t)(R_s - \hat{\mu}_t)', \tag{3.13}$$

where $\hat{\mu}_t = (\hat{\mu}_{1,t}, \hat{\mu}_{2,t}, \dots \hat{\mu}_{n,t})$, see (3.4).

We assume two parametric distributions for z_t: the standard Gaussian distribution and the standard Student-t distribution, each univariate or multivariate depending on whether the model is for $R_{p,t}$ or R_t. With the Student-t distribution we allow for fat tails in the distribution of z_t.

Following Mittnik and Paolella (2000), we use the weighted maximum likelihood (WML) procedure in order to give more weight to recent data, i.e. we multiply the log-likelihood contributions of the observation of period t by ρ^{T-t}, where ρ (≤ 1) is an exponential decay factor and T is the total number of observations used for estimation. If $\rho = 1$ we are back to usual ML estimation. We choose ρ as one minus the minimum of the failure rate (defined later in this section) for a given VaR confidence level.

Fig. 1 illustrates the failure rate-ρ relationship for portfolios made of EUR, GBP and JPY for an investor using VaR at 10% significance level. The model used to create this figure is the GARCH(1,1) with normal innovation distribution. The optimal ρ that minimizes the failure rate is equal to 0.994. We find similar results for other cases. Moreover, the value of the optimal ρ is robust to different innovation distributions. We use WML in an increasing window setup, i.e. the number of observations of the sample increases through time in order to consider the new information available. The improvement, in terms of better approximation to the desired confidence levels, using WML in an increasing window setup instead of ML, is of the order of 10%.

3.3 Estimation of the VaR

To estimate the VaR, we need to go back from the adjusted returns to the original ones. In the univariate case in which we work directly on the portfolio return, we simply multiply the estimated conditional means and standard deviations by the square root of the diurnal component $\phi(t)$ of the portfolio. The Value-at-Risk for time t at the confidence level α is calculated as

$$VaR^p_{t,\alpha} = (\hat{\mu}_{p,t} + \hat{\sigma}_{p,t}q_\alpha)\sqrt{\phi(t)}, \tag{3.14}$$

where $\hat{\mu}_{p,t}$ and $\hat{\sigma}_{p,t}$ are the forecasted conditional mean and standard deviation, and q_α is the (α)-th quantile of the distribution of z_t.

For the multivariate setting, once we have estimated the parameters of the model, we compute the forecast $\hat{\Sigma}_t$ of the matrix Σ_t using one of the multivariate GARCH models. After that, we introduce the diurnal factors by the following transformation:

$$\bar{\Sigma}_t = \text{diag}\left(\sqrt{\phi_1(t)}, \dots, \sqrt{\phi_n(t)}\right)\hat{\Sigma}_t \text{diag}\left(\sqrt{\phi_1(t)}, \dots, \sqrt{\phi_n(t)}\right)' \tag{3.15}$$

where $\phi_i(t)$, for $i = 1, \dots, n$, are the diurnal factors corresponding to each currency. With this matrix, we can compute the conditional variance of the portfolio return as

$$\bar{\sigma}_t^2 = w_t' \quad \bar{\Sigma}_t w_t, \tag{3.16}$$

where w_t is $n \times 1$ vector of portfolio weights. We also reintroduce the seasonality factor in each forecasted conditional mean: $\bar{\mu}_{i,t} = \hat{\mu}_{i,t}\sqrt{\phi_i(t)}$, for $i = 1, \dots, n$, where $\hat{\mu}_{i,t}$ is defined either as in (3.4) or, following (3.3), as $\hat{\mu}_i + \hat{\rho}_i R_{i,t-1}$ ($\hat{\mu}_i$ and $\hat{\rho}_i$ denoting the WML estimates). Thus, the forecasted conditional mean of the portfolio return is

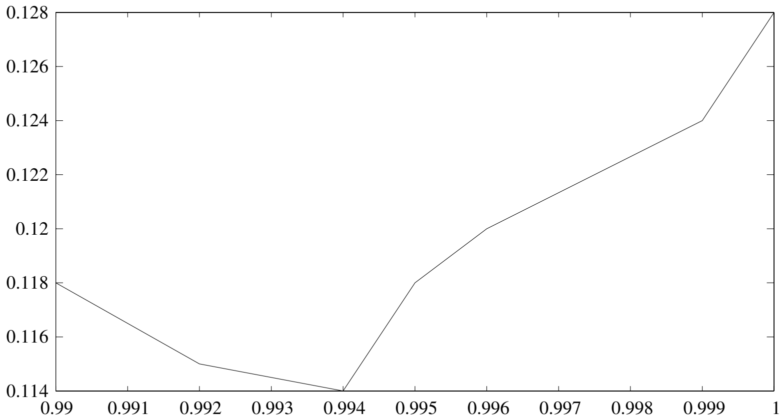
$$\bar{\mu}_t = w_t' \quad \hat{\mu}_t. \tag{3.17}$$

Once we have computed $\bar{\mu}_t$ and $\bar{\sigma}_t^2$, the Value-at-Risk for time t at the confidence level α is given by

$$VaR_{t,\alpha} = \bar{\mu}_t + \bar{\sigma}_t q_\alpha. \tag{3.18}$$

3.4 Determination of the optimal risky investment and amount to borrow or to lend

We compute the portfolio weights that maximize the expected return subject to the VaR constraint according to Eq. (2.14). Once we have determined the optimal weights for the investments in the risky currencies, and given the value of the desired VaR (VaR^*), we determine the amount borrowed or lent by using Eq. (2.15).



Failure rates (vertical axis) obtained with different ρ values (horizontal axis) using the geometric weighting scheme for a 30 out-of-sample days. Portfolios made of EUR, GBP, JPY and the USD is used as a numeraire for an investor's desired VaR significance level of $\alpha = 10\%$. The model used is the GARCH(1,1) with normal innovation distribution and an out-of-sample period from 11/17/2003 until 12/31/2003 (30 days).

Fig. 1. Failure rates-ρ relationship.

3.5 Evaluation of the models

We use four criteria to evaluate the models: two statistical tests and two economic tests. The first statistical test is the failure rate test proposed by Kupiec (1995). According to this test, the model is correctly specified if the observed portfolio return at time t is bigger than the VaR predicted at $t-1$ for time t in $100\alpha\%$ of the predictions. The failure rate for the long trading positions is defined as

$$f = \frac{1}{m} \sum_{t=T-m+1}^{T} \mathbf{1}[r_t < -VaR_{t,\alpha}], \tag{3.19}$$

where, m is the number of out-of-sample days, T is the total number of observations, r_t is the observed return at time t, $VaR_{t,\alpha}$ is the threshold value and $\mathbf{1}$ denotes the indicator function. Correspondingly, the failure rate for short trading positions is defined as the percentage of positive returns larger than the one-step-ahead VaR for short positions. Let η be the number of VaR violations in the out-of-sample interval of m points. Then, η has a binomial distribution with parameters α and m. Ideally, the failure rate should be equal to α. Thus, the null hypothesis is $H_0 : f = \alpha$. The corresponding likelihood ratio statistic,

$$LR = 2\log[f^{\eta}(1-f)^{m-\eta}] - 2\log[\alpha^{\eta}(1-\alpha)^{m-\eta}], \tag{3.20}$$

is asymptotically distributed as a $\chi^2(1)$ random variable.

A property that the VaR should have, besides respecting the VaR level, is that the VaR violations should not be serially correlated. In order to test this property, we use the dynamic conditional quantile test proposed by Engle and Manganelli (2004). The basic idea is that this property can be tested by defining the sequence

$$h_t = \mathbf{1}[r_t < -VaR_{t,\alpha}] - \alpha, \tag{3.21}$$

such that the expected value of h_t is zero. The dynamic quantile test is an (OLS) Fisher test under the null that all regression coefficients, including the intercept, are zero in a regression of the variable h_t on its own past, on the current VaR and on any other regressors. We perform the test using the current VaR and five lags of the VaR violations as explanatory variables.

The first economic test we use is based on the comparison of the wealth evolution provided by implementing the recommendations of the different models. Accordingly, the best model is the one that provides the highest wealth (or return) at the end of a forecast period, for a given risk level. The second economic test is based on the comparison of the results through a performance ratio. This performance ratio is defined as the ratio in (2.14), where the expected return is replaced by the realized return. With this ratio, we can compare the results of the different models. The preferred model is the one with the highest performance ratio.

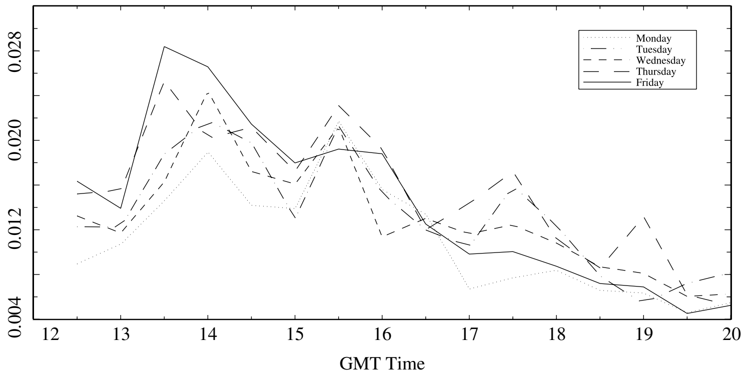
4. Empirical illustration

The foreign exchange market is a market maker based trading system, where dealers interact around the clock (i.e. in successive time zones). The most active trading centers are New York, London, Frankfurt, Sydney, Tokyo and Hong Kong. We

Table 1
Summary of models.

	Model name	Mean equation	Variance equations
Univariate			
	Emp-Emp	(3.4)	(3.5)
	Emp-GARCH	(3.4)	(3.6)
	AR-GARCH	(3.3)	(3.6)
Multivariate			
	Emp-DCC	(3.4)	(3.7)–(3.10)
	AR-DCC	(3.3)	(3.7)–(3.10)
	Emp-CCC	(3.4)	(3.11)
	AR-CCC	(3.3)	(3.11)
	Emp-BEKK	(3.4)	(3.12)
	AR-BEKK	(3.3)	(3.12)
	Emp-COV	(3.4)	(3.13)

Numbers in parenthesis denote equation numbers in the paper.



Each curve corresponds to one day of the week.

Fig. 2. Diurnal patterns of the EUR/USD volatility.

consider two portfolio allocation problems: one in which the dealer, located in the US, considers only two currencies (EUR and GBP) and one in which the dealer considers three currencies (EUR, GBP and JPY).

In this section, after describing the data, we present the portfolio recommendations of different econometric implementations, and the results of their statistical and economic evaluations. We programmed all computations using the Ox language, version 3.20 (see Doornik (2002)) and used a Pentium 4, 2.0 GHz. The average time to compute the optimal intradaily FX portfolio allocation depends on the model estimated. The most efficient one (Emp-CCC) takes around 2 min.

4.1. Data description

The database (provided by Olsen and Associates) consists of five-minute quotes for the EUR/USD, GBP/USD, and JPY/USD over the period ranging from January 1st, 1999 until December 31st, 2003, i.e. five years. These currency quotes are market makers' quotes and not transaction prices, as would be preferable. Since Danielsson and Payne (2002) showed that the statistical properties of five-minute US dollar/Deutsche Mark quotes are similar to those of transaction quotes, and transaction quotes are not widely available, we have resorted to using quotes. The database also contains the date, the time-of-day stamped to the five minutes in Greenwich mean time (GMT), and the mid-quotes.

From the five-minute mid-quotes, we compute thirty-minute and four hours returns, since we assume two cases where the dealers re-balance their portfolios every thirty minutes or four hours. The return at time t is computed as the difference between the logarithms of the mid-quotes at times t and $t-1$. We consider only data for the continuous trading period that goes from 12:00 GMT to 20:00 GMT (8 h per day). We exclude from the sample all the US holidays and control for daylight saving time (the time change between the winter and the summer). Finally, to avoid the trade opening noise, we eliminate the first return of the day. The total number of returns of our sample is equal to 20,144.

As explained in Section 3, we adjust the returns for the diurnal component of volatility. The seasonally adjusted (SA) returns are obtained by dividing the returns by the square root of their cross-sectional intradaily average volatility. Fig. 2 displays the intradaily diurnal functions for each day of the week for the EUR/USD, as an example.

Volatility is generally at its highest level one hour after the US market opening, due to the simultaneous activity of the American and the European markets. It decreases around 20 h GMT when the New York trading session ends. The profiles of the GBP/USD and USD/JPY diurnal functions are very similar to the ones shown in Fig. 2.

Table 2
Descriptive statistics of thirty-minute returns.

	EUR/USD		GBP/USD		JPY/USD	
	Returns	SA returns	Returns	SA returns	Returns	SA returns
Mean	0.002	0.025	0.002	0.030	−0.00005	0.002
SD	0.116	0.999	0.086	0.999	0.102	1.000
Max	1.115	9.707	0.788	9.667	0.994	9.756
Min	−0.845	−7.403	−0.562	−6.696	−1.343	−9.363
Skew	0.038	0.067	0.086	0.087	−0.345	−0.168
Kurt	8.558	7.429	6.667	6.008	13.250	8.900
ρ_1	−0.006	−0.016	−0.017	−0.026	−0.034	−0.041
ρ_2	0.013	0.008	0.009	0.006	0.005	0.003
$Q(1)$	0.69	5.09	5.51	13.48	23.32	34.12
	(0.105)	(0.024)	(0.019)	(0.0)	(0.0)	(0.0)
$Q(2)$	4.07	6.50	7.25	14.39	23.84	34.32
	(0.131)	(0.039)	(0.027)	(0.001)	(0.0)	(0.0)

The SA returns are the returns adjusted for the diurnal component of volatility (see Section 3.1). SD is the standard deviation, Skew and Kurt are the skewness and kurtosis coefficients, ρ_1 and ρ_2 the autocorrelation coefficients of order 1 and 2, $Q(1)$ and $Q(2)$ the corresponding Ljung–Box statistics, with their p-values below them (between brackets). The number of observations is 20,144 (period from 01/04/1999 until 12/31/2003).

Table 2 presents summary statistics of the thirty-minute returns for the three exchange rates, before and after seasonal adjustment. The means of the SA returns are almost equal to zero and their distributions have fatter tails than the normal, but they are almost symmetric. The distributions of the unadjusted returns are more leptokurtic and still close to being symmetric except for the JPY/USD. In the series of SA returns, there is a small but significantly (at the 2.5% level) negative autocorrelation of order one and a smaller positive autocorrelation of order two, which is not so significant. The negative autocorrelation in FX returns has been discussed in the academic literature. According to Goodhart and Figliuli (1991), the negative autocorrelation stems from constraints in the control of positions, while according to Bollerslev and Domowitz (1993) and Lo and MacKinlay (1990), this feature comes from the computation of asynchronous price series at the interval endpoints.

4.2. Example of investment recommendations

To illustrate our procedure, we present in detail the investment recommendations of a particular multivariate model. These recommendations specify the amount of US dollars to borrow or to lend and, if applicable, the allocation of this amount to each currency of the risky portfolio. The example is based on a portfolio of three risky currencies (EUR, GBP and JPY). The initial wealth is assumed to be one million USD. For the risk-free interest rate, we use the overnight rate in November 2003, equal to 4.47% (annually), which remains constant over the 30 day out-of-sample period. We consider two cases, and we assume that the dealers re-balance their positions every 30 min (16 times per day) and 4 h (2 times per day). At the end of the day, corresponding to the usual practice in FX trading, they close their positions, i.e. they buy (sell) the currencies on which they are short (long), and they lend the remaining USD at the overnight interest rate. However, the effect of this overnight rate has almost no impact on their wealth evolution.

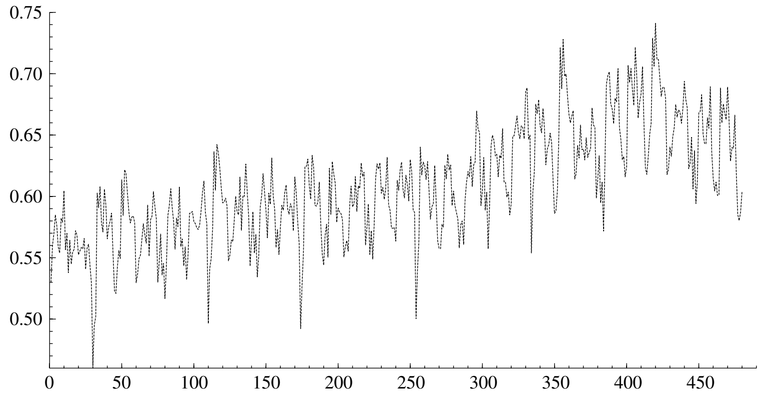
The particular model we consider for this example has a changing conditional mean vector estimated by the sample mean of the observations until time $t-1$.

For the conditional variance–covariance matrix, we use the BEKK specification of Engle and Kroner (1995) coupled with a trivariate Student-t-distribution and we estimate it using WML (setting ρ equal to 0.994, one minus the value that minimizes the failure rate for the specified confidence level). We fix the confidence level (α) for the estimation of the VaR at 10% and the desired VaR ($VaR*$) at 5%. This means that the VaR level associated with the risky portfolio allocation is smaller than the desired VaR, i.e. this position is less risky than desired. Accordingly, in order to obtain the desired VaR, the dealer should borrow a given amount of US dollars and invest it in the risky currencies. We use an estimation sample of 250 days (from 11/18/2002 until 11/16/2003), with 16 observations per day, and an out-of-sample forecasting period of 30 days (from 11/17/2003 until 12/31/2003).

Fig. 3 shows the evolution of the recommended amount of borrowed US dollars as a fraction of the initial wealth. For this specific example, and given that the assumed desired level of risk ($VaR*$) is larger than the portfolio VaR, the model always suggests to borrow at the risk-free rate and to invest the borrowed money in the risky currency portfolio. The average fraction of the dealer's wealth that is borrowed is equal to 61% over the forecast period, but there is clearly a positive trend in the fraction.

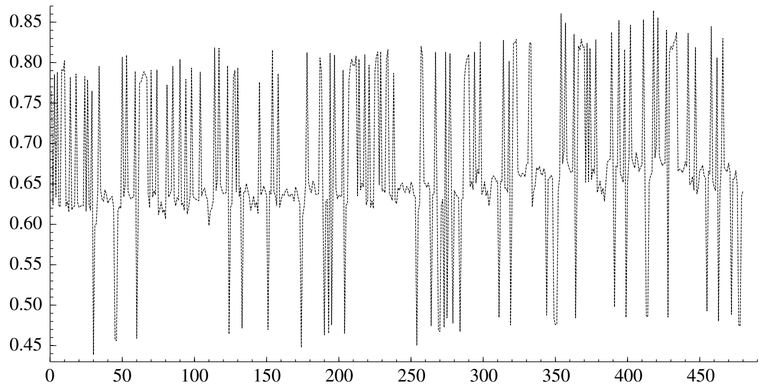
The model recommendations also concern the percentages of the funds (own and borrowed) to invest in each of the three risky currencies. Fig. 4 presents the weight evolution of one of the currencies, the euro. The weights fluctuate in three intervals of values: approximately 0.63–0.68 (most frequently), 0.75–0.80, and 0.45–0.50 (least frequently). On average, the model suggests investing 67% of the dealer's total funds in EUR. The corresponding averages for GBP and JPY are 58% and 36%, respectively.

These model recommendations are consistent with the return evolution of the currencies during the out-of-sample period. Table 3 presents the descriptive statistics for this period. The average return of the EUR/USD is almost 44% and



This figure displays the fraction of wealth to be borrowed over 30 trading days of 8 hours, with 30-minutes re-balancing, for the out-of-sample forecast period from 11/17/2003 until 12/31/2003. Portfolios made of three currencies: EUR, GBP and JPY. Model: changing mean and BEKK specification with a Student-t distribution.

Fig. 3. Recommended fraction of wealth to borrow.



This figure displays the recommended evolution of the euro share over 30 trading days of 8 hours, with 30-minutes re-balancing, for the out-of-sample forecast period from 11/17/2003 until 12/31/2003. Portfolios made of three currencies: EUR, GBP and JPY. Model: changing mean and BEKK with a Student-t distribution.

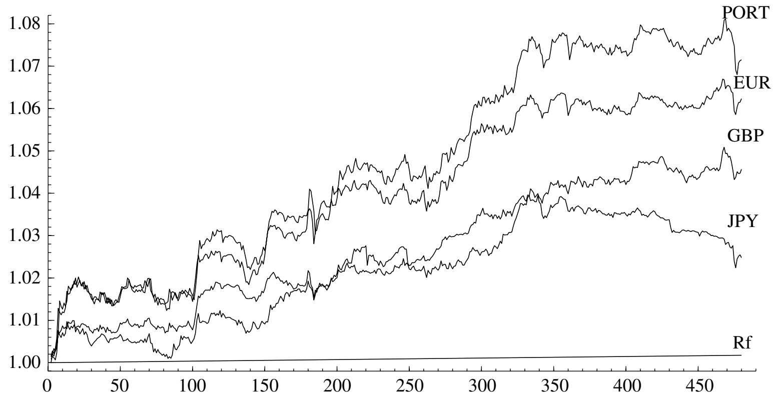
Fig. 4. Evolution of the euro weight in the portfolio.

Table 3
Descriptive statistics of the forecast period returns (Sample 1, 30 min).

	EUR/USD	GBP/USD	JPY/USD
Mean	0.013	0.009	0.005
SD	0.108	0.079	0.080
Max	0.704	0.524	0.311
Min	−0.510	−0.275	−0.447
Skew	0.515	0.811	−0.517
Kurt	8.005	6.989	7.002
ρ_1	0.080	0.003	−0.035
ρ_2	−0.042	−0.030	−0.027
Q (1)	3.11	0.003	0.56
	(0.078)	(0.958)	(0.450)
Q (2)	3.95	0.39	0.98
	(0.139)	(0.822)	(0.610)

SD is the standard deviation, Skew and Kurt are the skewness and kurtosis coefficients, ρ_1 and ρ_2 the autocorrelation coefficients of order 1 and 2, Q (1) and Q (2) the corresponding Ljung–Box statistics, with their p-values below them (between brackets). The number of observations is 480, corresponding to the forecast period from 11/17/2003 until 12/31/2003. The data frequency is 30 min.

160% larger than for the GBP/USD and the JPY/USD, respectively. Therefore, it is not surprising that the dealer takes more position in EUR and in GBP than in JPY, as recommended by the model.



This figure displays the wealth evolution of investing the initial wealth at the risk-free interest rate (Rf), in single currency portfolios (EUR, GBP or JPY), and in optimal portfolios (PORT) of the three currencies for the 30-day out-of-sample forecast period from 11/17/2003 until 12/31/2003 (with 8 hours trading per day and 30-minutes re-balancing). Model: changing mean and BEKK specification with a Student-t distribution.

Fig. 5. Wealth evolution of five investment strategies.

Finally, Fig. 5 presents the wealth evolution of five investment strategies for an initial wealth of one million dollars. Three strategies correspond to investments in a single currency at a time (EUR, GBP, JPY), one corresponds to the optimal portfolio of the three currencies (PORT) derived from the model, and the last one (Rf) consists of investing the initial wealth at the risk-free interest rate. It emerges firstly that the riskless strategy (Rf) generates a smaller final wealth than the other strategies. Secondly, the wealth obtained by the intradaily optimal portfolio allocation model is always larger than the wealth obtained by investing only in GBP or in JPY, and most of the time also larger than the wealth obtained by investing only in EUR. As already pointed out, the results can be explained by the observation that, during the forecast period, the returns generated by the euro are, most of the time, positive.

4.3. Economic and statistical evaluation

In this section, we present the economic and statistical evaluation of the models presented in Section 2. The economic evaluation criteria are the total return (R) and the performance ratio (PR) at the end of the investment period (i.e. an out-of-sample forecast period). The statistical criteria are the failure rate test (FR) and the dynamic quantile test (Dq). These criteria and tests are explained in detail at the end of Section 3.

In Table 4 we present the results for portfolios made of two currencies (EUR and GBP), and in Table 5 the results for portfolios made of three currencies (EUR, JPY, and GBP). In both tables, we consider thirty-minutes portfolio rebalancing. The out-of-sample period goes from 11/17/2003 until 12/31/2003 (30 trading days). For the failure rate test, we report the test statistic, and values in bold indicate significance at the 5% level. For the Dq test, we report the p-value of the F-statistic. From these results we draw the following conclusions:

1. Most failure rate tests are significant at the 5% level for multivariate models, except the ‘Emp-COV’ model, but the reverse is true for univariate models. However, we observe that the models are conservative, since most of the failure rates are below the desired α.
2. Almost all the models pass the dynamic quantile test at the 5% significance level, the exceptions being a few univariate models for two currencies. It appears that most models are correctly specified in the sense that the VaR violations are not serially correlated.
3. Compared to the Student-t distribution, the normal one produces, in all cases, a higher return and performance ratio when $\alpha = 1\%$, while the reverse conclusion is true in most cases at 5% and especially at 10%.
4. The returns obtained by estimating the conditional mean using the empirical mean of the data until time $t - 1$ are in a large majority of cases not smaller than the returns obtained by estimating the mean using an autoregressive process. For the performance ratios, no specification of the mean dominates the other. In terms of the statistical tests, the two approaches deliver similar results.
5. The results obtained by the BEKK and the DCC models are very similar but it takes almost 50% more CPU time to use the BEKK model than the DCC model.
6. The results obtained by the BEKK and DCC models dominate in most cases those obtained by the CCC model. Thus, the time-varying correlations seem to matter in our portfolio optimization setting.
7. The returns and performance ratios obtained by the ‘Emp-Emp’ and the ‘Emp-COV’ models are almost as good as those obtained using the univariate and multivariate GARCH models. Nevertheless, the ‘Emp-Emp’ and the ‘Emp-COV’ models

Table 4
Evaluation criteria of models for two currencies (Sample 1, 30 min, EUR and GBP).

Models		$\alpha = 1\%$		$\alpha = 5\%$		$\alpha = 10\%$	
		Normal	Student	Normal	Student	Normal	Student
	R	0.055	0.046	0.078	0.081	0.098	0.105
	PR	12.43	9.48	26.28	27.89	44.19	52.28
Emp-Emp	FR	0.023	0.020	0.081	0.085	0.117	0.135
	Dq	**0.03**	0.94	0.33	0.07	0.79	0.57
	R	0.061	0.047	0.087	0.078	0.109	0.110
	PR	9.49	8.02	20.00	23.19	35.71	42.61
Emp-GARCH	FR	0.004	**0.012**	0.019	0.017	0.038	0.046
	Dq	0.54	0.62	**0.03**	0.06	0.42	0.07
	R	0.050	0.045	0.071	0.075	0.095	0.092
	PR	9.01	10.68	17.45	19.89	28.74	48.56
AR-GARCH	FR	**0.010**	**0.008**	0.029	0.029	0.065	**0.087**
	Dq	0.99	0.93	0.71	**0.04**	059	**0.01**
	R	0.054	0.034	0.079	0.057	0.101	0.079
	PR	2.57	2.68	8.05	8.45	12.59	13.16
Emp-CCC	FR	**0.013**	**0.010**	0.031	0.035	0.067	**0.085**
	Dq	**0.01**	**0.01**	**0.05**	**0.03**	0.14	0.09
	R	0.036	0.028	0.051	0.059	0.062	0.080
	PR	3.46	2.59	6.49	6.06	12.15	11.99
AR-CCC	FR	0.013	0.010	0.031	0.035	0.068	0.082
	Dq	**0.02**	**0.00**	0.06	**0.05**	0.09	0.07
	R	0.047	0.037	0.065	0.061	0.081	0.094
	PR	4.01	2.66	7.48	8.00	12.06	16.10
Emp-DCC	FR	**0.017**	**0.010**	**0.054**	**0.056**	**0.096**	**0.116**
	Dq	0.29	0.99	0.31	0.62	0.23	0.47
	R	0.037	0.031	0.051	0.061	0.065	0.085
	PR	3.11	2.48	6.28	9.08	14.37	16.28
AR-DCC	FR	**0.015**	**0.008**	**0.048**	**0.042**	**0.079**	**0.081**
	Dq	0.99	0.99	0.23	0.29	0.06	0.91
	R	0.043	0.040	0.059	0.070	0.080	0.094
	PR	3.12	2.89	7.07	8.87	12.15	16.46
Emp-BEKK	FR	**0.017**	**0.010**	**0.050**	**0.052**	**0.081**	**0.104**
	Dq	0.99	0.99	0.10	0.40	0.18	0.34
	R	0.041	0.038	0.086	0.070	0.100	0.091
	PR	3.18	3.01	9.86	9.57	14.15	17.91
AR-BEKK	FR	**0.015**	**0.010**	**0.040**	0.031	0.060	0.069
	Dq	0.99	0.99	0.09	0.05	0.58	0.55
	R	0.058	0.052	0.079	0.084	0.100	0.108
	PR	13.25	10.24	27.11	27.99	45.38	54.82
Emp-COV	FR	0.019	0.021	0.074	0.081	0.110	0.121
	Dq	0.046	0.72	0.43	0.12	0.31	0.64

This table presents the statistic and economic criteria for evaluating the models over the out-of-sample 30-day forecast period from 11/17/2003 until 12/31/2003. The optimization procedure is carried out each 30 min, i.e. 16 re-balances per day. R denotes the total return of the investment over the 30-day period. PR is the performance ratio computed according to Eq. (2.14), where the expected return is replaced by the actual return. FR is the empirical failure rate, with bold numbers indicating significance at the 5% level. Dq is the p-value of the F-statistic for the dynamic quantile test (see Section 3). For a definition of the models, see Table 1.

have higher failure rates, which indicates that it is not as correctly specified and that the risk is higher than the target one.

8. Figures, displayed in Tables 6 and 10, obtained with four hour rebalancing and corresponding to three currency portfolio exhibits slightly better results as those corresponding to thirty minute rebalancing. The difference comes mostly from the lower transaction costs paid in case of four hour rebalancing.

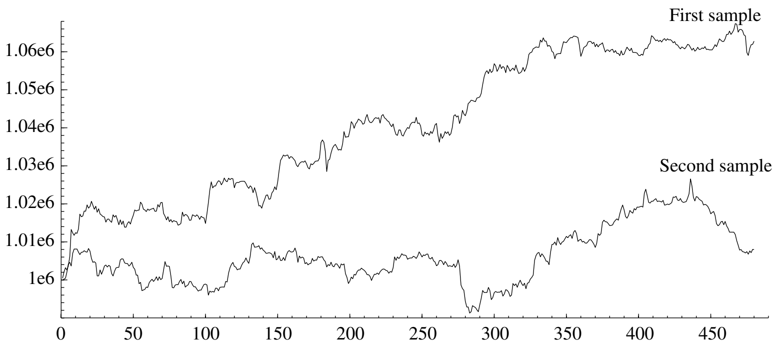
Bauwens et al. (2006) state that whether "the univariate repeated approach is more adequate than the multivariate one" is an open question. According to our results, using a multivariate GARCH model (either the CCC, BEKK or the DCC) provides better results in the statistical and economic sense. Moreover, the multivariate approach is more economical in CPU time than the repeated univariate one: for example, the computing time is reduced by 33% for two currencies and by 50% for three when using the DCC model, compared to a univariate GARCH model (since the latter has to be estimated many times, when searching for the optimal weights of the portfolio).

In order to ensure that the results presented above are not driven by a sequence of 'abnormal' positive returns for some currency, we compute a second set of portfolio allocations for another sample. The estimation period goes from 06/19/2002 until 06/17/2003 and the out-of-sample forecast period from 06/18/2003 until 07/31/2003 (30 days). Fig. 6 shows the wealth evolution in both samples if everything is invested in EUR. Table 7 presents the descriptive statistics of the second sample.

Table 5
Evaluation criteria of models for three currencies (Sample 1, 30 min).

Models		$\alpha = 1\%$		$\alpha = 5\%$		$\alpha = 10\%$	
		Normal	Student	Normal	Student	Normal	Student
	R	0.031	0.026	0.043	0.044	0.057	0.058
	PR	1.46	1.05	2.99	3.06	5.12	5.82
Emp-Emp	FR	0.027	0.023	0.094	0.106	0.113	0.144
	Dq	0.48	0.98	0.38	0.48	0.73	0.97
	R	0.024	0.024	0.038	0.040	0.048	0.051
	PR	3.15	2.15	3.97	4.75	7.01	8.75
Emp-GARCH	FR	0.005	**0.009**	0.039	**0.043**	**0.082**	**0.089**
	Dq	0.41	0.23	0.14	0.23	0.56	0.38
	R	0.019	0.014	0.039	0.038	0.046	0.047
	PR	1.99	1.58	6.12	5.87	6.42	7.45
AR-GARCH	FR	0.003	**0.012**	0.039	0.041	**0.091**	**0.094**
	Dq	0.13	0.09	0.51	0.07	0.47	0.12
	R	0.027	0.024	0.037	0.036	0.058	0.062
	PR	15.13	13.11	24.56	26.89	42.01	43.25
Emp-CCC	FR	0.015	0.006	**0.046**	0.540	**0.100**	0.127
	Dq	0.08	0.08	0.12	0.09	0.12	0.11
	R	0.024	0.024	0.027	0.034	0.046	0.045
	PR	4.08	6.18	17.13	18.87	33.16	35.58
AR-CCC	FR	0.001	**0.007**	0.002	0.001	0.005	0.007
	Dq	0.19	0.21	0.07	0.06	0.15	0.17
	R	0.024	0.022	0.042	0.047	0.058	0.069
	PR	14.12	11.00	29.58	30.45	49.42	54.85
Emp-DCC	FR	**0.010**	**0.008**	**0.045**	**0.043**	**0.089**	**0.093**
	Dq	0.99	0.99	0.32	0.35	0.37	0.42
	R	0.024	0.022	0.039	0.040	0.055	0.064
	PR	13.12	13.05	24.78	25.19	49.87	61.24
AR-DCC	FR	**0.008**	**0.007**	**0.044**	**0.047**	0.081	**0.086**
	Dq	0.99	0.99	0.26	0.36	0.78	0.31
	R	0.028	0.020	0.041	0.044	0.060	0.068
	PR	15.01	10.87	30.15	30.87	62.02	74.56
Emp-BEKK	FR	**0.017**	**0.008**	0.033	**0.043**	**0.091**	**0.094**
	Dq	0.99	0.99	0.73	0.67	0.25	0.09
	R	0.027	0.017	0.042	0.041	0.051	0.057
	PR	13.25	11.09	30.57	36.12	58.33	70.41
AR-BEKK	FR	**0.008**	**0.008**	0.035	0.034	**0.091**	**0.099**
	Dq	0.95	0.94	0.72	0.65	0.12	0.09
	R	0.032	0.027	0.044	0.045	0.059	0.062
	PR	1.49	1.17	3.12	3.01	5.34	6.03
Emp-COV	FR	0.018	0.019	0.077	0.082	0.112	0.130
	Dq	0.12	0.45	0.23	0.16	0.30	0.52

This table presents the statistic and economic criteria for evaluating the models over the out-of-sample 30-day forecast period from 11/17/2003 until 12/31/2003. The optimization procedure is carried out each 30 min, i.e. 16 re-balances per day. R denotes the total return of the investment over the 30-day period. PR is the performance ratio computed according to Eq. (2.14), where the expected return is replaced by the actual return. FR is the empirical failure rate, with bold numbers indicating significance at the 5% level. Dq is the p-value of the F-statistic for the dynamic quantile test (see Section 3). For a definition of the models, see Table 1.



First sample from 11/17/2003 until 12/31/2003, second from 06/18/2003 until 07/31/2003.

Fig. 6. Wealth evolution of EUR portfolios for two different samples.

Table 6
Evaluation criteria of models for three currencies (Sample 1, 4 h).

Models		$\alpha = 1\%$		$\alpha = 5\%$		$\alpha = 10\%$	
		Normal	Student	Normal	Student	Normal	Student
	R	0.035	0.032	0.046	0.045	0.051	0.054
	PR	2.14	2.01	3.16	3.78	4.72	5.25
Emp-Emp	FR	0.032	0.032	0.087	0.0728	0.146	0.123
	Dq	0.15	0.21	0.16	0.23	0.24	0.32
	R	0.031	0.027	0.039	0.041	0.049	0.055
	PR	2.00	1.97	2.97	3.45	4.89	5.32
Emp-GARCH	FR	0.015	0.019	0.041	**0.045**	**0.096**	**0.099**
	Dq	0.19	0.27	0.34	0.29	0.42	0.38
	R	0.028	0.027	0.036	0.039	0.048	0.051
	PR	1.89	1.86	2.78	3.28	4.74	5.04
AR-GARCH	FR	0.013	**0.011**	**0.045**	**0.048**	**0.091**	**0.093**
	Dq	0.16	0.21	0.14	0.23	0.45	0.57
	R	0.029	0.029	0.039	0.041	0.050	0.052
	PR	22.35	21.34	28.75	33.57	55.90	63.2
Emp-CCC	FR	0.018	0.021	0.057	0.059	**0.098**	**0.102**
	Dq	0.07	**0.05**	0.06	0.08	0.13	0.15
	R	0.028	0.027	0.035	0.039	0.049	0.053
	PR	19.99	18.93	25.44	30.01	50.03	57.89
AR-CCC	FR	**0.012**	0.018	**0.057**	0.065	0.110	**0.102**
	Dq	**0.02**	**0.00**	0.09	0.16	0.43	0.36
	R	0.029	0.024	0.044	0.046	0.054	0.057
	PR	21.23	24.19	29.98	37.53	54.48	65.93
Emp-DCC	FR	**0.011**	**0.009**	**0.046**	**0.048**	**0.099**	**0.101**
	Dq	0.45	0.56	0.87	0.78	0.82	0.91
	R	0.027	0.025	0.039	0.045	0.049	0.052
	PR	19.42	24.87	28.76	36.04	49.06	56.85
AR-DCC	FR	**0.012**	**0.010**	**0.048**	**0.051**	0.089	0.091
	Dq	0.23	0.12	0.27	0.34	0.38	0.45
	R	0.030	0.028	0.043	0.049	0.057	0.069
	PR	24.20	20.14	31.40	39.82	54.98	69.91
Emp-BEKK	FR	**0.010**	**0.009**	**0.047**	**0.050**	**0.096**	**0.097**
	Dq	0.99	0.99	0.85	0.82	0.87	0.75
	R	0.028	0.027	0.041	0.040	0.055	0.064
	PR	22.37	20.01	29.48	29.96	53.10	63.37
AR-BEKK	FR	**0.008**	**0.009**	**0.049**	**0.049**	**0.098**	**0.101**
	Dq	0.94	0.95	0.76	0.78	0.22	0.17
	R	0.036	0.029	0.048	0.045	0.053	0.057
	PR	2.17	1.97	3.25	3.95	4.88	5.33
Emp-COV	FR	0.029	0.031	0.082	0.071	0.111	0.129
	Dq	0.14	0.15	0.32	0.19	0.09	0.23

This table presents the statistic and economic criteria for evaluating the models over the out-of-sample 30-day forecast period from 11/17/2003 to 12/31/2003. The optimization procedure is carried out each 4 h, i.e. 2 re-balances per day. R denotes the total return of the investment over the 30-day period. PR is the performance ratio computed according to Eq. (2.14), where the expected return is replaced by the actual return. FR is the empirical failure rate, with bold numbers indicating significance at the 5% level. Dq is the p-value of the F-statistic for the dynamic quantile test (see Section 3). For a definition of the models, see Table 1.

Table 7
Descriptive statistics of the forecast period returns (second sample).

	EUR/USD	GBP/USD	JPY/USD
Mean	0.002	0.005	0.004
SD	0.103	0.090	0.084
Max	0.391	0.314	0.316
Min	−0.497	−0.441	−0.589
Skew	0.056	−0.171	−0.434
Kurt	5.654	5.076	8.916
ρ_1	0.115	−0.010	−0.067
ρ_2	−0.075	−0.003	−0.030
Q(1)	6.35	0.043	2.16
	(0.012)	(0.84)	(0.142)
Q(2)	8.14	0.046	2.47
	(0.017)	(0.97)	(0.291)

SD is the standard deviation, Skew and Kurt are the skewness and kurtosis coefficients, ρ_1 and ρ_2 the autocorrelation coefficients of order 1 and 2, $Q(1)$ and $Q(2)$ the corresponding Ljung–Box statistics, with their p-values below them (between brackets). The number of observations is 480, corresponding to the forecast period from 06/18/2003 until 07/31/2003. The data frequency is 30 min.

Table 8
Evaluation of models for two currencies (Sample 2, 30 min, EUR and GBP).

Models		$\alpha = 1\%$		$\alpha = 5\%$		$\alpha = 10\%$	
		Normal	Student	Normal	Student	Normal	Student
Emp-Emp	R	0.013	0.012	0.018	0.018	0.024	0.025
	PR	5.14	4.57	11.85	12.99	21.12	26.10
	FR	0.021	0.019	0.068	0.075	0.133	0.131
	Dq	0.49	0.53	0.66	0.32	0.12	0.10
Emp-Garch	R	0.011	0.009	0.016	0.014	0.019	0.022
	PR	31.14	29.75	9.05	9.98	15.47	22.01
	FR	0.003	0.006	0.071	**0.052**	**0.101**	**0.100**
	Dq	0.20	0.05	0.06	0.56	0.21	0.05
AR-Garch	R	0.010	0.008	0.012	0.015	0.015	0.019
	PR	23.10	19.99	9.87	10.11	15.01	21.45
	FR	0.004	0.007	**0.053**	**0.055**	**0.089**	**0.094**
	Dq	0.08	0.09	0.12	0.41	0.38	0.29
Emp-CCC	R	0.010	0.010	0.011	0.012	0.017	0.020
	PR	8.45	9.05	11.25	11.04	13.12	14.48
	FR	0.040	0.025	0.077	0.089	**0.106**	0.133
	Dq	**0.03**	**0.01**	0.06	**0.04**	0.12	0.10
AR-CCC	R	0.010	0.009	0.012	0.011	0.016	0.019
	PR	9.01	8.29	10.82	11.21	12.53	12.67
	FR	0.037	0.025	0.075	0.084	0.112	0.132
	Dq	**0.03**	**0.00**	0.07	**0.04**	0.09	0.07
Emp-DCC	R	0.010	0.009	0.014	0.015	0.020	0.021
	PR	11.03	10.95	12.58	12.86	14.52	15.69
	FR	**0.010**	**0.011**	**0.052**	**0.049**	**0.089**	**0.101**
	Dq	0.15	0.41	0.26	0.28	0.29	0.41
AR-DCC	R	0.009	0.008	0.012	0.014	0.017	0.019
	PR	9.42	8.12	10.49	11.58	13.00	15.93
	FR	0.005	**0.007**	**0.054**	**0.051**	**0.085**	**0.095**
	Dq	0.05	0.38	0.56	0.36	0.19	0.35
Emp-BEKK	R	0.010	0.010	0.014	0.015	0.022	0.023
	PR	11.12	11.04	12.97	13.06	15.18	16.07
	FR	**0.009**	**0.008**	**0.050**	**0.051**	**0.090**	**0.097**
	Dq	0.05	0.07	0.51	0.48	0.35	0.09
AR-BEKK	R	0.009	0.008	0.013	0.013	0.019	0.019
	PR	10.89	11.06	13.18	13.76	14.27	15.91
	FR	**0.008**	**0.007**	**0.046**	**0.045**	**0.087**	**0.090**
	Dq	0.12	0.09	0.46	0.35	0.26	0.12
Emp-COV	R	0.014	0.012	0.020	0.021	0.025	0.026
	PR	5.21	4.67	12.03	13.20	21.83	26.91
	FR	0.021	0.018	0.061	0.075	0.128	0.131
	Dq	0.12	0.26	0.32	0.19	0.44	0.17

This table presents the statistic and economic criteria for evaluating the models over the out-of-sample 30-day forecast period from 06/18/2003 to 07/31/2003. The optimization procedure is carried out each 30 min, i.e. 16 re-balances per day. R denotes the total return of the investment over the 30-day period. PR is the performance ratio computed according to Eq. (2.14), where the expected return is replaced by the actual return. FR is the empirical failure rate, with bold numbers indicating significance at the 5% level. Dq is the p-value of the F-statistic for the dynamic quantile test (see Section 3). For a definition of the models, see Table 1.

Comparing the statistics in Tables 7 and 3, we see that the returns for the second 30-day forecast sample are smaller than for the first one, especially for the EUR/USD, while the standard deviations are about the same. Moreover, during the second period, the currency that has the largest mean return is the GBP, and not the EUR. If we look at Tables 8 and 9, we see that the forecasted returns (R) are considerably smaller than those obtained for the first out-of-sample period (see Tables 4 and 5). These differences are consistent with the smaller returns observed during the second period. However, we draw the same conclusions as from the first forecasting experience: the multivariate GARCH models perform better than the univariate GARCH model; the Student distribution gives better results than the normal when α increases; the 'Emp-Emp' and 'Emp-COV' models yield returns that are about the same as those provided by the multivariate GARCH models, but it has higher failure rates, and so on.

We construct a benchmark built on a 'naive strategy' that consists in making an equally weighted portfolio that we keep unchanged during the thirty-days out-of-sample period. The total return triggered by the latter strategy is shown in Table 11. The 'naive' returns are, in general, higher than the ones obtained using univariate and multivariate GARCH models in the case where the traders are highly risk averse ($\alpha = 1\%$). However, the 'naive' returns are almost below the returns obtained by mid-level risk averse traders ($\alpha = 5\%$), and they are always below the returns generated by low risk averse traders ($\alpha = 10\%$).

Table 9
Evaluation criteria of models for three currencies (Sample 2, 30 min).

Models		$\alpha = 1\%$		$\alpha = 5\%$		$\alpha = 10\%$	
		Normal	Student	Normal	Student	Normal	Student
Emp-Emp	R	0.014	0.013	0.020	0.020	0.019	0.020
	PR	9.48	7.25	19.59	20.97	23.14	28.00
	FR	0.038	0.029	0.075	0.079	0.126	0.133
	Dq	0.07	0.04	0.25	0.29	0.09	0.12
Emp-GARCH	R	0.009	0.008	0.011	0.012	0.014	0.015
	PR	54.65	42.85	14.18	19.42	20.02	26.87
	FR	0.006	0.003	**0.049**	**0.050**	**0.095**	**0.096**
	Dq	0.44	0.23	0.71	0.72	0.27	0.48
AR-GARCH	R	0.008	0.005	0.010	0.010	0.013	0.015
	PR	45.48	40.54	18.29	18.84	22.12	30.43
	FR	0.007	0.001	**0.046**	**0.052**	**0.094**	**0.095**
	Dq	0.26	0.38	0.75	0.79	0.31	0.45
Emp-CCC	R	0.011	0.010	0.017	0.020	0.021	0.024
	PR	7.18	6.57	14.19	17.16	26.51	31.05
	FR	0.018	0.02	**0.057**	0.063	0.120	0.124
	Dq	0.07	0.06	0.09	0.07	0.15	0.13
AR-CCC	R	0.009	0.008	0.014	0.016	0.017	0.020
	PR	6.42	6.12	13.58	12.45	23.13	26.73
	FR	0.019	0.021	0.067	0.063	0.14	0.12
	Dq	**0.04**	**0.03**	0.07	0.06	0.09	0.06
Emp-DCC	R	0.015	0.014	0.020	0.023	0.021	0.023
	PR	8.17	6.26	16.29	17.69	27.47	34.53
	FR	**0.016**	**0.010**	**0.048**	**0.054**	**0.094**	**0.094**
	Dq	0.22	0.05	0.05	0.09	0.15	0.27
AR-DCC	R	0.014	0.013	0.018	0.017	0.018	0.020
	PR	7.08	5.78	17.69	17.93	30.18	37.17
	FR	0.005	**0.007**	**0.045**	**0.049**	**0.092**	**0.096**
	Dq	0.32	0.09	0.16	0.21	0.24	0.38
Emp-BEKK	R	0.016	0.014	0.021	0.024	0.018	0.023
	PR	6.51	7.10	14.21	17.58	23.71	31.39
	FR	**0.013**	**0.016**	0.038	**0.052**	**0.091**	**0.093**
	Dq	0.09	0.35	0.23	0.10	0.58	0.37
AR-BEKK	R	0.015	0.013	0.018	0.018	0.017	0.019
	PR	8.75	6.14	15.02	18.19	27.07	35.77
	FR	**0.014**	**0.016**	0.035	0.041	**0.087**	**0.090**
	Dq	0.10	0.29	0.25	0.18	0.45	0.29
Emp-COV	R	0.015	0.013	0.021	0.021	0.021	0.021
	PR	10.07	7.49	20.30	20.98	24.11	31.02
	FR	0.028	0.025	0.072	0.074	0.116	0.130
	Dq	0.23	0.52	0.38	0.71	0.43	0.81

This table presents the statistic and economic criteria for evaluating the models over the out-of-sample 30-day forecast period from 06/18/2003 to 07/31/2003. The optimization procedure is carried out each 30 min, i.e. 16 re-balances per day. R denotes the total return of the investment over the 30-day period. PR is the performance ratio computed according to Eq. (2.14), where the expected return is replaced by the actual return. FR is the empirical failure rate, with bold numbers indicating significance at the 5% level. Dq is the p-value of the F-statistic for the dynamic quantile test (see Section 3). For a definition of the models, see Table 1.

Finally, we compare our econometric models in a mean–variance setting. The performance measure used for the issue is the Sharpe ratio displayed in Table 12. We notice that 'Emp-Emp' and 'Emp-COV' models almost outperform the conditional variance models, and the multivariate model surpass the univariate one. This result is not surprising and confirms the findings of Giannopoulos et al. (2005) and Zhang and Huang (2006) according to which the empirical variance underestimates the risk. By considering conditional variance models, the multivariate GARCH models yield higher Sharpe ratio than univariate ones, and the highest performance ratio is triggered by the 'Emp-BEKK'model.

5. Conclusion

We design and implement a portfolio selection model which allocates a portfolio of currencies by maximizing the expected return subject to VaR constraint. Based on an econometric implementation using intradaily data, we compute the optimal portfolio at regular time intervals during a sequence of trading days. For the estimation of the conditional variance from which the VaR is computed, we use the standard univariate GARCH model of Bollerslev (1986), and three multivariate GARCH models, the CCC model of Bollerslev (1990), the BEKK model of Engle and Kroner (1995), and the DCC model of Tse and Tsui (2002). We evaluate the models using two economic criteria and two statistical tests. The procedure we have

Table 10
Evaluation criteria of models for three currencies (Sample 2, 4 h).

Models		$\alpha = 1\%$		$\alpha = 5\%$		$\alpha = 10\%$	
		Normal	Student	Normal	Student	Normal	Student
Emp-Emp	R	0.018	0.014	0.017	0.019	0.020	0.022
	PR	11.43	9.45	21.38	24.78	26.71	29.40
	FR	0.026	0.025	0.073	0.073	0.131	0.135
	Dq	**0.03**	**0.01**	0.14	0.19	0.12	0.15
Emp-GARCH	R	0.011	0.011	0.013	0.012	0.016	0.017
	PR	44.01	42.15	25.16	19.08	26.14	31.01
	FR	0.012	0.001	**0.051**	**0.051**	**0.011**	**0.099**
	Dq	0.54	0.46	0.62	0.63	0.37	0.27
AR-GARCH	R	0.010	0.008	0.010	0.012	0.014	0.016
	PR	22.14	19.08	21.19	20.05	23.94	28.14
	FR	**0.009**	0.002	0.042	**0.048**	**0.096**	**0.098**
	Dq	0.42	0.41	0.56	0.63	0.22	0.31
Emp-CCC	R	0.013	0.011	0.021	0.023	0.025	0.027
	PR	7.18	6.97	17.25	19.27	28.19	33.12
	FR	**0.011**	0.005	0.059	**0.054**	0.131	0.128
	Dq	**0.04**	**0.03**	0.08	0.06	0.12	0.16
AR-CCC	R	0.011	0.009	0.017	0.018	0.018	0.021
	PR	6.57	6.00	14.12	13.37	22.18	27.14
	FR	0.017	0.019	0.072	0.061	0.139	0.123
	Dq	**0.04**	**0.00**	0.14	0.14	0.08	0.12
Emp-DCC	R	0.016	0.014	0.024	0.025	0.021	0.023
	PR	7.96	7.01	17.85	18.57	26.32	31.49
	FR	**0.012**	**0.011**	**0.049**	**0.052**	**0.097**	**0.098**
	Dq	0.26	0.14	0.17	0.19	0.55	0.42
AR-DCC	R	0.014	0.013	0.017	0.016	0.019	0.021
	PR	7.15	6.92	17.36	18.03	28.07	34.19
	FR	**0.012**	0.001	**0.046**	**0.047**	**0.094**	**0.098**
	Dq	0.34	0.43	0.18	0.25	0.42	0.33
Emp-BEKK	R	0.016	0.014	0.022	0.023	0.021	0.022
	PR	7.57	7.12	14.42	19.95	25.34	32.38
	FR	**0.011**	**0.013**	0.042	**0.047**	**0.093**	**0.093**
	Dq	0.12	0.23	0.21	0.22	0.48	0.35
AR-BEKK	R	0.014	0.013	0.019	0.018	0.018	0.019
	PR	6.25	6.21	15.36	17.71	24.48	31.94
	FR	0.016	**0.014**	0.041	**0.044**	**0.092**	**0.091**
	Dq	0.14	0.36	0.42	0.36	0.46	0.35
Emp-COV	R	0.020	0.017	0.019	0.021	0.021	0.024
	PR	12.03	9.78	22.04	23.49	27.14	30.22
	FR	0.023	0.022	0.070	0.071	0.129	0.134
	Dq	0.12	0.06	0.27	0.08	0.19	0.07

This table presents the statistic and economic criteria for evaluating the models over the out-of-sample 30-day forecast period from 06/18/2003 to 07/31/2003. The optimization procedure is carried out each 4 h, i.e. 2 re-balances per day. R denotes the total return of the investment over the 30-day period. PR is the performance ratio computed according to Eq. (2.14), where the expected return is replaced by the actual return. FR is the empirical failure rate, with bold numbers indicating significance at the 5% level. Dq is the p-value of the F-statistic for the dynamic quantile test (see Section 3). For a definition of the models, see Table 1.

Table 11
Naive strategy returns.

Portfolio	Sample 1	Sample 2
EUR and GBP	0.055	0.015
EUR, GBP and JPY	0.045	0.016

This table presents the total return of the investment (over 30 day-period) from implementing a naive strategy for a portfolio made of 2 and 3 currencies, respectively. Sample 1 goes from 11/17/2003 to 12/31/2003 and Sample 2 from 06/18/2003 to 07/31/2003.

developed could be a useful tool to help foreign exchange dealers to control the accomplishment of a daily VaR level while maximizing their profits.

Our results show that estimating the VaR using the conditional variance and from multivariate GARCH models improves the results of the optimal portfolio allocation, respectively, compared to using the unconditional variance or a univariate model. There is no substantial difference between the results obtained by using the BEKK and the DCC models, but the latter is preferable since it takes much less computing time. However, both outperform the CCC model. Thus, the time-varying correlations seem to matter in our portfolio optimization setting. The Student-t distribution performs better than the normal when the risk level used to define the VaR is large (5% or 10%), while the reverse holds at 1%.

Table 12
Sharpe ratio (Sample 1).

Evaluation freq.	30 min		4 h	
	Normal	Student	Normal	Student
Emp-Emp	0.94	0.91	0.91	1.02
Emp-GARCH	0.56	0.42	0.58	0.65
AR-GARCH	0.41	0.34	0.54	0.57
Emp-CCC	0.84	0.81	0.94	0.96
AR-CCC	0.78	0.74	0.89	0.90
Emp-DCC	0.88	0.84	0.94	1.01
AR-DCC	0.75	0.71	0.83	0.87
Emp-BEKK	0.92	0.88	0.95	1.04
AR-BEKK	0.86	0.83	0.88	0.89
Emp-COV	0.98	0.92	0.94	1.18

This table presents the Sharpe ratio for evaluating the mean–variance model over the out-of-sample 30-day forecast period from 11/17/2003 to 12/31/2003. The currency portfolio involves EUR, GBP and JPY. The optimization procedure is carried out each 30 min and each 4 h, i.e. 16 and 2 re-balances per day, respectively.

Future research would include, among others, the study and implementation of time varying re-balancing periods and the analysis of its implications in portfolio allocations. In terms of econometric techniques, future research can be conducted to test other multivariate GARCH models and potential competitors to these kinds of models, such as the Wishart Autoregressive Process (WAR) model proposed by Gourieroux et al. (2004). The latter is a dynamic model for stochastic volatility matrices. An interesting property of this model is that it is invariant with respect to the choice of the numeraire. In the GARCH framework, the extension to high dimensions is known to increase the number of parameters heavily, so that applying our methodology to more than a handful of currencies may seem a priori difficult. However, the recent work by Engle et al. (2008) about the estimation of large dimensional GARCH models indicates that the extension to high dimensions is feasible.

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